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Designing the World's Best Badminton Racket

Ph.D. Thesis

by

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Preface

This thesis is submitted in partial fulfillment of the requirements for the Doctor of Philosophy at the Faculties of Engineering, Science and Medicine. The work has been carried out during the period from June 2007 to June 2010 under the supervision of Professors John Rasmussen and Ole Thybo Thomsen at the Department of Mechanical and Manufacturing Engineering, Aalborg University.

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Finally, I would like to acknowledge all the members of the AnyBody Group for their eagerness in helping me to overcome obstacles in many different aspects of the project, and for making the office a truly enjoyable place to work.

Abstract

From its humble beginnings in wood to its current slim geometry in high-quality lightweight carbon fiber composites, improvements in the badminton racket have largely been due to material advancements. The link between design and performance remains relatively poorly understood. Current rackets are designed heuristically, based on experience of the manufacturer and the player. The objective of this thesis is to add a scientific perspective to the design of badminton rackets, by studying the underlying physics of the game.

The design of a badminton racket requires an understanding of the player, the racket, and how they interact in generating a stroke. The dynamics of the stroke can be used to assess their performance. Performance can be characterized in terms of power and control, which can respectively be quantified by the shuttlecock speed and the consistency of the stroke. Since the stroke involves both rigid-body and flexible-body dynamics of the racket, it can be characterized by several stroke parameters: racket head speed at impact from the rigid-body motion, and elastic deflection and elastic velocity at impact from the deformation behavior. We find that motion capture at a frame rate of 500 Hz and strain gage recordings at 2400 Hz are adequate for reliable measurements of the rigid-body and flexible-body dynamics, respectively, of the racket during the stroke. A promising new device consisting of accelerometers and gyroscopes is also presented, as an alternative to motion capture.

The effects of the racket's mechanical properties on the dynamics of the stroke can be summarized by the decrease in racket head speed with increasing racket moment of inertia and the reduction in stroke consistency with increasing racket compliance. Stroke consistency also decreases with increasing racket moment of inertia, and power can increase slightly with greater racket compliance. In performance terms, this translates to lower power and lower control with a racket of large moment of inertia, and lower control and slightly higher power with a more flexible racket.

Some effects of the skill level of the player are also observed. Advanced players exhibit higher racket head speeds at impact and generally greater stroke consistency. The high impact head speed can largely be attributed to the high peak angular velocity, not observed in recreational players.

Based on these results, a scheme for finding the optimal racket swingweight and stiffness for a given player is proposed. Maximum shuttlecock speed is the first indicator of optimal swingweight. For some players, shuttlecock speeds are similar over a range of swingweights. Minimum stroke variation can then be applied to determine the ideal swingweight, as well as optimal racket stiffness.

Publications

Parts of this work have been published or submitted for publication.

Publications in refereed journals

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- Kwan, M., de Zee, M., and Rasmussen J. Effects of racket swingweight on racket power in badminton. (Submitted May 2010 to *Journal of Sports Sciences*)

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Contents

1

Introduction

It doesn't take a racket scientist to figure out that no single racket is the "world's best racket". Different players have different playing styles, and therefore prefer different rackets. The difficulty lies in finding an explanation for these preferences, and ultimately to start customizing the racket to the player. Currently, badminton rackets are largely developed using heuristic methods based on player intuition and experience, and subsequently evaluated by player feedback. While this trial-and-error method can eventually lead to well-performing rackets, the design process could be improved and streamlined by an analytical approach. Designing a badminton racket requires both recognizing what performance characteristics are desired, as well as understanding how different racket properties will affect the performance. Basically, good design requires knowing what to achieve, and how to achieve it.

Desirable qualities in a racket can be summed up by power and control. Performance characteristics can be deduced by understanding the game of badminton, the winning strategies and tactics used by expert players. A few statistical studies on various aspects of rallies during tournaments have been conducted, concluding that net, drop and block shots are the most effective return strokes (Tong and Hong, 2000), and smash is the most effective kill shot (Lee et al., 2005; Tong and Hong, 2000). This leads back to the concepts of power and control, where smash is a power stroke and net shots require more finesse and technical control over the racket.

To understand the influence of the racket on performance, we should analyze (1) the interaction between the racket and the shuttlecock, and (2) the interaction between the racket and the player. The racket-shuttlecock interaction can be examined from a mechanical point of view, as a collision between two objects. The racket-player interaction is more complex, involving both the mechanical aspects of the racket and the biomechanical aspects of the player. Dynamics of the stroke are influenced primarily by the racket's mass properties and the player's skill and strength. The deformation behavior of the racket is in turn influenced by the dynamics of the stroke as well as stiffness properties of the racket. The key to good racket design is understanding how all these interactions ultimately relate to performance. Literature on the badminton racket is rather slim, but studies involving other sporting implements, especially tennis rackets, could prove to be relevant.

1.1 Impact of racket and ball

The mechanics of impact has great significance in many sports, including tennis. Many tennis studies focus on modeling the collision between the racket and ball, using principles of momentum and energy conservation.

The collision between racket and ball can be represented by a rigid body model, as a collision between a rod and a particle (Brody, 1997). The complexity of modeling the string bed is generally avoided by representing the interaction between the ball and strings with a coefficient of restitution (COR). The COR is defined by

$$\mathrm{COR} = \frac{v_2 - V_2}{V_1 - v_1} \tag{1.1}$$

where v and V are the velocities of the ball and racket impact point, respectively. Pre-impact and post-impact are indicated by 1 and 2, respectively.

For tennis rackets, the variation of the measured COR (typically around 0.85) over the racket head is not large, so a constant COR value can be used to approximate outgoing ball speed (Brody, 1997). The apparent coefficient of restitution (ACOR) can then be calculated. The ACOR is the ratio of rebound ball speed to incident ball speed, ACOR $= v_2/v_1$, sometimes referred to as the ball speed ratio, e (Brody, 1997; Cross, 1999).

The ACOR can also be determined experimentally, or through a flexible body model of the ball-racket collision. In a flexible body model developed by Cross (1999), the racket is represented by a series of finite beam elements, and the ball is represented by a simple spring. This model accounts for vibrational losses of the racket post-impact as well as losses due to ball hysteresis. An added benefit is that deflection behavior of the racket can also be modeled.

To determine the ACOR experimentally, the appropriate boundary conditions must first be determined. The question about whether the handheld racket should be represented as a free, clamped or semi-clamped racket, or whether it is dependent on grip firmness has been studied extensively (Brody, 1995, 1997; Cross, 1998, 2004, 1999). The answer is dependent on whether the hand plays a significant role in the dynamics of the racket and ball during collision. One indication of the appropriate boundary conditions is the post-impact vibrational frequencies. Even for a very tight grip, vibration frequencies of the handheld racket were much closer to the free racket (100-200 Hz) than to the clamped racket (25-30 Hz) (Brody, 1995). Beam vibration frequency is only reduced slightly from 193 to 171 Hz (free to handheld) (Cross, 2004). Since the vibration frequency is not greatly affected by the hand, it would seem a free racket is the better model.

The influence of the hand can also be assessed by comparing wave propagation times and ball dwell time. Ball contact induces a transverse wave that travels along the racket which is then reflected at the ends of the racket. If the wave arrives back at the impact location after the ball has left the strings, then the influence of the hand force is inconsequential to the exit velocity of the ball. For tennis, dwell time of the ball is about 8 ms (Brody, 1997; Cross, 2004). In the handheld case, the measured wave propagation time from the impact point to the handle was 1.5 ms, indicating that the hand does impose a force on the handle while the ball in still in contact (Cross, 1998). However, the hand force can generally be neglected, since the reflected wave usually does not reach the impact point before the ball has left (Cross, 2004). In addition, Cross (1999) showed that measurements of ACOR (for a sufficiently long beam) are unaffected by boundary conditions. This is seemingly contradictory to the common belief between coaches and players that a firm grip is essential, but these results simply indicate that the hand force has no effect on the outgoing ball speed. A firm grip may be necessary for other reasons, such as maintaining racket speed during the swing leading into impact or preventing injury during impact.

Most of these findings can be applied directly to modeling the impact between a badminton racket and shuttlecock. Both the rigid body and flexible body models are suitable for any collision between a long slender body and a point mass. The cork nose of the shuttlecock is considerably stiffer than a tennis ball, so elastic deformation effects will likely play a less significant role in the impact dynamics (Silva et al., 2005). Given the approximately elliptical shape of the racket head in tennis and badminton, the COR can also be considered nearly constant across the string bed of a badminton racket, although the value of the COR could be different, dependent on string tension. Vibrational behavior at impact of any handheld racket can be represented by a freely supported beam, although wave propagation times and ball dwell time may differ from those in tennis. For estimating exit velocity of the ball, however, boundary conditions are unimportant for impact away from the ends.

The study of impact also has applications in the analysis of the sweet spots of the racket. These sweet spots can be defined as: the center of percussion, the vibration node, and the location of the maximum ACOR (Cross, 1998). The first two definitions involve the dynamic response of the racket post-impact, the shock and vibration that must be absorbed by the arm. In tennis, this is of particular interest to injury prevention and to design of a more comfortable racket. The third definition pertains to the spot on the racket head that leads to the greatest outgoing ball velocity. Designing a racket such that these three points coincide, or nearly coincide, can improve the “feel” of the racket, by maximizing racket power while minimizing shock and vibration to the hand.

1.2 Stroke dynamics and racket mass properties

Stepping back a little, we can also look at the swing leading up to impact and see how it is affected by different racket properties. This is the interaction between the player and the racket, how the racket influences the stroke of the player. The relationship of most interest is the one between the mass properties of the racket and the stroke speed.

The effects of the mass properties on the stroke have been studied in several sports, including tennis (Brody, 2000; Mitchell et al., 2000) and baseball (Fleisig et al., 2002; Nathan, 2003). Mass properties of the racket or bat are commonly characterized by the total mass and the moment of inertia (MOI) about the relevant swing axis, often referred to as “swingweight”. Results of a player sensitivity analysis show that proficient tennis players can distinguish differences as small as 2.5% in moment of inertia and around 5% in the polar moment of the racket (Brody, 2000). This high degree of sensitivity suggests that racket moment of inertia plays an important role in the performance of the racket. Several studies in tennis and baseball have confirmed that swingweight has a strong correlation with swing speed, whereas swing speed is seemingly independent of total mass.

In a study on the performance of the tennis serve, Mitchell et al. (2000) found that a smaller racket MOI could significantly increase the racket head speed for skilled players. The relationship between swing speed and swingweight was later modeled by Cross (2001), who proposed a power law of the form $\omega = k/I_o^n$. Swing speed is represented by the angular velocity ω, I_o is the swingweight about the swing axis near the handle of the racket, k is a constant dependent on player strength, and n is the exponent that dictates the relationship between swingweight and swing speed for a given player. A value of $n = 0.5$ implies the total energy of the racket $(0.5I_o\omega^2)$ remains constant across swingweight, and $n = 0$ means the constant total energy of the arm is constant. If the total power input is constant, then $n = 0.33$ (Nathan, 2003). From experimental results of a tennis serve, a good fit to the data was found to be $\omega = 28.64/I_o^{0.314}$ (Cross, 2001). A value of $n = 0.314$ suggests that the swingweight-swing speed relationship is driven by a constant (or near constant) power input.

Similar studies have been conducted with bats. Using baseball and softball bats with different masses and MOIs, Fleisig et al. (2002) found a linear relationship between bat sweet spot linear velocity and bat MOI, but no significant correlation between bat linear velocity and bat mass. Although originally correlated with a linear fit, the swingweight-swing speed data was later fitted with the power law, arriving at an approximate value of $n = 0.3$ (Nathan, 2003). Smith et al. (2003) performed a controlled study on baseball bats of differing MOIs while keeping mass constant and vice versa. Again, swing speed was shown to be strongly correlated with bat swingweight with $n = 0.25$, but nearly independent of bat total mass. For a group of players of varying skill level, the values of n ranged from 0.08 to 0.37, but no consistent trend was observed between n and player skill.

For tennis rackets and baseball bats, the swing speed is nearly independent of the total mass, while the relationship between swing speed and swingweight can be described by the power law, with typical values of n around 0.3. For badminton rackets, however, these relationships have not yet been investigated. Although baseball bats and tennis rackets are in a significantly heavier mass and swingweight range than badminton rackets, the same principles may be applicable. Cross and Bower (2006) investigated the swing speeds over a large range of swingweight, including a lighter swingweight comparable to badminton rackets. Using "general striking implements", i.e.

rods made of brass, aluminum, and wood, a swingweight range of 0.0103 to 0.1034 kg-m^2 (constant mass: 0.321 kg) was tested. They found that $n = 0.12$ for lower swingweights and $n = 0.27$ for higher swingweights. However, with only three rods to test the entire span of swingweights, the values of n are only rough estimates.

From the studies above, the relationship between swingweight and swing speed has been investigated through the transfer of energy from racket to ball. The energy transfer from the arm to the racket has also been studied. Using a double pendulum to represent the motion of the arm with a racket, Cross (2005) showed the importance of the relative masses of the arm and racket in maximizing the swing speed of the racket. He also determined that for the best energy efficiency, the racket mass should be approximately equal to the geometric mean of the arm mass and the ball mass. In tennis, the ratio between the masses of the arm (2000 g), racket (340 g), and ball (57 g) is about 5.9 times. In baseball, the ratio is about 6.4-two thick arms (6000 g), bat (930 g), and ball (144 g). This pattern also appears true for badminton. For a shuttlecock at 5 g, a racket at 95 g, and an arm at 1850 g, the ratio is about 19.

1.3 Stroke dynamics and racket compliance

In addition to the mass properties of the racket, the compliance also plays a role in the dynamics of the stroke. During the swing, the racket is subjected to high accelerations which generate inertial loads on the racket, causing the racket to bend.

The deflection behavior of a racket during a stroke can be modeled after a rotating flexible beam (Montagny et al., 2003). The focus of the study by Montagny et al. (2003) was on the development of a non-linear flexible beam model to predict the deflection behavior of the beam in planar motion, subsequently validated through experimental measurements of three different badminton rackets. However, model analysis was used as a means of model validation rather than deformation measurements, so no information on racket deflection behavior was provided.

Racket deformation during the stroke has not been studied. Most of the literature concerning tennis racket stiffness is aimed towards its effects during the time of impact. In addition, tennis rackets are considerably stiffer that badminton rackets, since the energy that goes into racket deformation is not returned to the ball (Brody, 1997). Deflection behavior during the stroke is therefore probably negligible. Studies on the deformation of golf shafts, however, could prove to be useful.

Studies on significance of golf shaft stiffness are somewhat contradictory. One study concludes that shaft flexibility plays only a minor dynamic role in the golf swing, dispelling the myth that shaft elasticity can provide an increase in impact speed (Milne and Davis, 1992). However, another study found significant differences in clubhead speed with varying shaft stiffnesses for all eight golfers tested (Worobets and Stefanyshyn, 2007). They also observe that the golfers produced their highest clubhead speed with different clubs, indicating that it could be possible to match the appropriate shaft stiffness to a given player. It is widely believed by manufacturers and golfers alike that shaft stiffness is important, but both studies found that the players were generally unable to distinguish between the different stiffnesses. This puzzling paradox, however, is perhaps best left to the golf scientists to figure out. Based on the deformation of the golf shaft presented by Milne and Davis (1992), it can be seen that the large end mass on the golf club results in a profoundly different dynamic behavior from that of a badminton racket. Golf shaft stiffness studies are therefore of limited relevance to the dynamics of the badminton racket.

1.4 Project definition and approach

The main goal of this project is to study the dynamics of the stroke in order to understand how the performance of the stroke is influenced by properties of the racket. This knowledge can be used for aiding a player find the ideal racket, or for developing new racket designs. The approach to the project is summarized in the following steps:

1. Characterize racket (racket properties).
2. Identify measurable quantities to characterize stroke dynamics (stroke parameters).
3. Vary racket properties and measure stroke parameters.
4. Identify stroke parameters that correlate to performance factors.
5. Optimize racket properties to maximize performance.

The racket must first be characterized. Mechanical properties of the racket can be divided into mass properties and stiffness properties. Geometry of the racket could also be an important factor in optimization, e.g. for minimizing drag or maximizing the area of the sweet spot, but these aspects are not considered here. However, a simple quick analysis of the drag forces on the racket is detailed in Appendix A. This study mainly concerns the racket's mass and stiffness properties and their effects on the dynamics of the stroke. Since important variables cannot yet be distinguished from less significant variables, as many properties as possible are measured, including total mass, center of mass, moment of inertia, radius of gyration, flexural modulus, torsional stiffness, and fundamental frequency. By doing so, we hope to identify which have the most significant influences on the stroke.

The influence of the racket on the stroke is studied by analyzing the dynamics of the stroke, which involves both the kinematics and the deformation of the racket. Several parameters can be defined to characterize the stroke, e.g. racket head speed, duration of the stroke, peak deflection, etc. The next step is to determine how these stroke parameters translate into "racket quality". By analyzing and comparing several groups of data, we try to find trends that can help identify which stroke parameters are useful for evaluating performance.

In the badminton community, performance factors known as "power" and "control" are often used to describe a racket. These two terms are somewhat abstract, referring to the "feel" of the racket in the player's hand rather than the mechanical definition of power or the theoretical concept of control. To transform these terms into more concrete values, we need to find stroke characteristics to describe power and control. Power is the most directly translatable: greater power leads to higher shuttlecock speeds. Higher shuttlecock speeds come from more momentum transferred from the racket to the shuttle. Momentum transfer is dependent on both the mass distribution of the racket and the racket velocity at impact. Quantifying control is more complex. Control describes how accurately the racket can be moved, or how precisely the shuttlecock can be placed. This is likely a very complicated aspect involving sensory feedback from the racket-hand connection to the player, but a possible solution is to simply measure the variation in the stroke, assuming greater variations indicate less control. Another performance consideration is nimbleness, or how quickly a stroke can be made. This can be assessed by measuring the duration of the stroke.

The last step is to find the racket properties that give the optimal performance. Ideally, this would involve maximizing all the performance factors: power and control. However, performance factors are generally in competition, so an optimal performance profile needs to be determined. Once the performance criteria are established, the optimization process can begin.

The analysis of racket dynamics here focuses on a single stroke technique: the smash. The assumption is that the results from these tests could also be carried over to other strokes. That is, if the performance of the smash stroke could be optimized with a certain set of racket properties, this same set of racket properties would also result in a well-performing racket for the other strokes.

1.5 Outline of thesis

The remainder of the thesis is outlined as follows:

Chapter 2 is an overview of the experimental methods used throughout the project and the challenges faced. Different methods are compared, and a concept for a new measuring device is introduced.

Chapter 3 presents the first measurements of the elastic deformation of a badminton racket during the stroke, which are then applied to evaluation stroke consistency of elite players.

Chapter 4 compares different methods of measuring racket deflection, and a beam model relating inertial loads and deflections is developed.

Chapter 5 explores the possible advantages of measuring racket head speed as an alternative to shuttlecock speed to evaluate performance, including indications of differences between player skill level and technique through racket kinematics.

Chapter 6 investigates the relationship between racket moment of inertia and racket head speed as well as shuttlecock speed.

Chapter 7 is a collection of observations on the role of elasticity in stroke performance, and its dependence on player skill level.

Chapter 8 summarizes the findings of this project, culminating in a proposed procedure for identifying the optimal racket properties for a given player. Recommendations for future work are also discussed.

Appendix A provides an idea of the effects of racket geometry on drag forces during the stroke.

Appendix B details the calculations involved in using a new device to measure racket motion, consisting of accelerometers and gyroscopes.

2

Experimental Work

In the search for the most important design parameters of a badminton racket, a good dose of modeling and a great deal of experiments are involved. The main focus is on studying the overall dynamic response, so the model of the racket is kept rather simple, as simple as possible while still capable of giving useful results. Material and manufacturing aspects of racket design are not investigated, so a finite element model of the small details and complexities of the racket geometry is not necessary. However, it could be useful to develop such a model for predicting frequency modes and deformation behavior of rackets of varying stiffness properties and mass distributions.

2.1 Measuring racket properties

The badminton racket is composed of several parts: butt cap, handle, top cap, shaft, heart, frame and string bed, labeled in Fig. 2.1.

For modeling purposes, the racket can be divided into three main segments: handle, shaft, and head (frame+string bed). Each segment is approximated as a uniform beam, where the shaft and head segments were modeled as

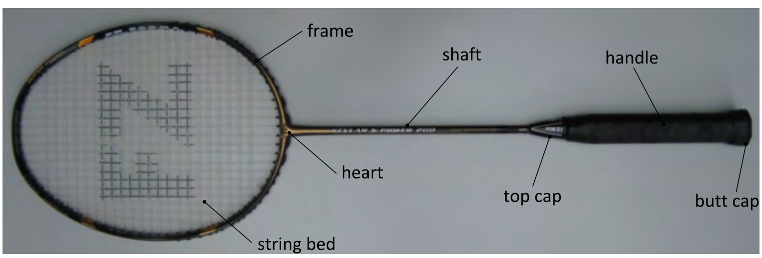

Figure 2.1: Anatomy of the racket.

flexible beams, and the handle was assumed to be completely rigid. The big challenge lies in determining how to accurately measure mass and stiffness properties of each of the three racket segments, without destroying the racket. We would like to keep the racket intact for subsequent testing with players.

Segment mass properties can be roughly estimated based on data sheets provided by the manufacturers. The accuracy of these segment masses are then assessed by comparing calculated and measured whole racket properties, including center of mass. However, small discrepancies lead to significant margins of error, since the racket itself is so lightweight.

Obtaining segment stiffness properties is also challenging. Cantilever bending tests with a series of different weights are performed to determine the flexural modulus of the shaft and head segments. In these tests, the handle is rigidly clamped, but the exact location of the fixed point is difficult to pinpoint due to the complexities of the racket in this area-the tapered geometry of the wooden handle and the top cap, as well as the "joint" between the shaft and handle. Since the shaft is simply glued into the handle, the rigidity of this attachment can vary from racket to racket. The effective fixed point location can therefore be unique for every racket. The stiffness of the head segment is dependent on string tension, which is not studied here. Despite these obstacles, some estimates of the racket stiffness can be found. However, there is no guarantee that stiffnesses obtained from static tests are relevant in a dynamic situation.

Given its long list of drawbacks, the segmented model was abandoned, and a uniform beam model representing the entire racket is considered instead. While a single uniform beam is an extreme simplification of the racket, a uniform beam is a good representation of the shaft, where the majority of the bending occurs. The simplicity of the model is also appealing, requiring only two inputs to characterize the racket: fundamental frequency and beam length. The fundamental frequency of the racket is measured with much greater certainty than the segmental stiffnesses required for the previous model. In addition, racket frequency is a characteristic of the dynamic response, in contrast to the segment stiffnesses derived from static deflection tests. The challenge with this model is determining the appropriate beam length, i.e. transforming a non-uniform beam into a uniform one. By applying a known loading function and measuring the deflection profile, the uniform beam length can be calculated.

A simple, effective method for determining these racket properties is through strain measurements. Strain gages can be used to find racket frequency with high accuracy, sensitive even to changes in string tension. Effective beam length can also be derived from strains, but requires determining the fixed point location, which can be estimated from static deflection tests. With these racket parameters as inputs, the beam model can be applied to transform strain measurements to deflections of the racket.

2.2 Measuring racket motion

Badminton is claimed to be the world's fastest racket sport, with shuttlecock speeds reaching over 330 km/h (Chi, 2005). While this makes for a sensational statistical achievement, it also leads to many challenges in making measurements with sufficiently high sampling rates and resolution. In addition, impact often leads to sharp discontinuities in the data, which lead to difficulties in filtering.

The stroke can be divided into several phases: backswing, forward swing, contact, and follow-through. These phases are illustrated in Fig. 2.2. While contact time is relatively well-defined, the divisions between the other phases are less obvious. In our definition, the transition from backswing to forward swing occurs when angular velocity crosses from negative to positive. The start and end times of the entire stroke are left undefined.

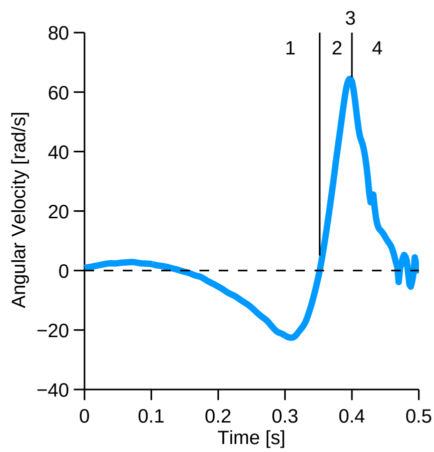

Figure 2.2: Phases of the stroke: (1) backswing, (2) forward swing, (3) contact, and (4) follow-through.

2.2.1 Measurements with high speed video

Our first measurements were done with high speed video, a quick way to visually capture the behavior of the racket during a stroke. High speed video eliminates the problem of sufficient sampling rate, but requires a great deal of lighting to sample at very high rates. The markers used should simply be visually high-contrast, so interference with the racket is minimal, but the image processing involved in tracking these markers can be extremely tedious. In addition, having only a single camera reduces the dimension of the stroke to a planar perspective, which introduces distance errors. Precise measurements of the racket's dynamic behavior are therefore not possible with this approach.

Figure 2.3: High speed video frame captured at 2000 fps.

2.2.2 Measurements with strain gages

Strain gages offer a simple solution to measuring the elastic behavior of the racket during the stroke. Sampling rate is not a problem. Strain gages provide nice clean signals, with low noise and high resolution and minimal interference with the racket. However, the wires running down the handle and the player's arm are a minor annoyance. The wires also restrict the player's range of motion, since the player is essentially tethered to the measurement unit.

Proper alignment of the strain gages on the racket shaft can sometimes be problematic, but the biggest difficulty is calibration, determining the scaling factor from strains to deflections. An analytical model of a uniform cantilever beam is used to relate strain and head deflection, requiring input parameters from the racket that are not so straightforward to measure. These issues are discussed in Chapter 4.

Figure 2.4: Experimental set-up of racket instrumented with strain gages.

Another disadvantage is that strain measurements provide very limited information about the stroke kinematics. While deflections can be calculated directly from strains, the model can only return a loading function which is related to the accelerations of the stroke. This loading function is essentially composed of linear and angular accelerations, forming an equivalent angular acceleration, α_E. With only a single equation, the linear and angular components of α_E cannot be separated with strain gage data alone.

2.2.3 Measurements with motion capture

Motion capture provides a complete overview of the racket dynamics during the stroke, including linear kinematics, angular kinematics, and deflections. However, accuracy and reliability of this information can be problematic due to sampling rate and resolution limitations, especially since differentiation is required to calculate velocity and acceleration data. In our experience, reliable kinematic data can be obtained recording at the system maximum of 500 fps, followed by an optimized post-processing routine (Andersen et al., 2009). Deflection behavior can be observed to some extent, but measurements are generally quite noisy compared with calculated values from strains. The extremely high speeds reached during a smash stroke also cause problems for the markers themselves, especially those attached at the head. For elite players, tape is completely inadequate; markers need to be tied onto the racket head. Zip-ties work with reasonable success, but we find that wire is the best solution. Unfortunately, this adds considerable weight to the racket head, leading to some not-so-insignificant changes in the mass properties of the racket.

Figure 2.5: Attachment of reflective markers to racket head using zip-ties.

Shuttlecock speed can also be measured with motion capture, but is complicated by the fact that shuttlecock velocity decays rapidly following the impact. An accurate velocity profile of the shuttlecock is difficult to obtain, due to the extremely high magnitudes paired with a motion capture system working at its limit of 500 Hz. It seems logical to define an average shuttlecock speed measured a short time after the impact, but there does not seem to be a consensus in the literature on how to measure shuttlecock speed.

2.2.4 Measurement solutions

To determine the full dynamic behavior of the racket during the stroke, we find that the best solution is a combination of motion capture and strain gages. Racket kinematics are measured using motion capture, and strains are better for measuring elastic behavior, as well as finer details of stroke motion like variation around impact. Using the beam model, deflections are calculated from strains, and then checked with deflection magnitudes measured from motion capture. The beam model could also be used to predict the deflection behavior given the normal acceleration profile, calculated from racket kinematics. However, the transition from α_E to deflections involves a convolution, so it is necessary to have very clean acceleration input, which is not possible with motion capture.

The problem of sharp discontinuities in the data resulting from impact is difficult to resolve. Depending on the objective, different filtering schemes can be taken. A quick and dirty solution is to filter over just a section of the data, from the beginning of the stroke until just before impact. The fidelity of the data leading into impact is then preserved, by preventing the undesirable smoothing that results from filtering through the impact. However, this can lead to strange end effects from the filter and a discontinuity between the sections of filtered and unfiltered data. The best solution may be to avoid filtering altogether, by measuring velocities or accelerations directly, thereby eliminating or reducing the need for differentiation. Another solution is to avoid impact by not providing a target to hit. However, we observe that the player tends to reach lower maximum speeds without a target to aim for, which is likely a psychological effect. Another observation about the players, especially advanced players, is that they usually prefer that the shuttlecock is thrown rather than hanging. A moving target may therefore be better suited for advanced players, but for novice players, it may introduce an increased degree of variation in the stroke that masks the effects being looked into.

2.3 Developing a racket motion sensor

The primary motivation behind developing a motion sensor for the racket is to sidestep the inconveniences of using an elaborate camera system. Since we are focused on the kinematics of the racket only, measuring the velocities and accelerations directly with a small device attached to the racket would be much simpler than going through the time-consuming ordeal of marker position data processing first. Other benefits include high sampling rate for improved accuracy and portability, so players would not need to come into a laboratory setting. Ideally, the system could even be used in a game situation.

The first concept involves the use of two triaxial accelerometers. However, a sufficiently high range triaxial accelerometer is difficult to find; racket accelerations for a badminton stroke performed by an elite player can exceed 50 g. An alternative is to combine single- and dual-axis accelerometers, but this setup introduces irreconcilable orientation and alignment errors. This problem can be resolved by placing most of the accelerometer ICs on a single circuit board to ensure alignment, as proposed in Meamarbashi (2009). However, this solution requires having the resources to design and fabricate a printed circuit board.

Another idea is to combine a triaxial accelerometer with a triaxial gyroscope. With linear accelerations of a point on the racket and angular velocities of the racket, the complete kinematics of the racket can be calculated. The disadvantage to using a self-standing device attached to the racket rather than a camera system is the loss of orientation data, which relates the local and global coordinate systems. Racket orientation can however be calculated if the racket is initially at rest. Some additional processing is also necessary to remove gravity effects from the acceleration data. Details of these calculations are found in Appendix B.

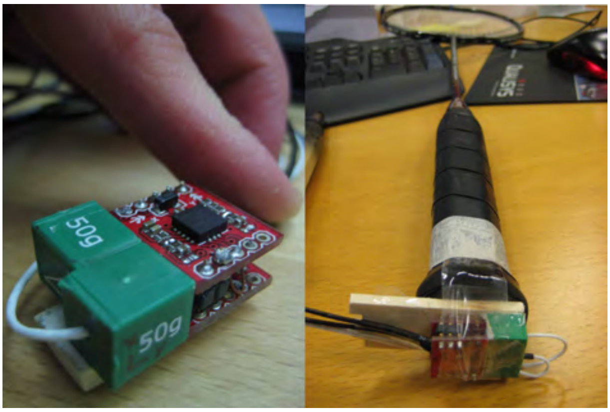

Figure 2.6: Racket motion sensor: accelerometers and gyroscopes.

3

Article 1

Kwan, M. and Rasmussen, J. (2010). The importance of being elastic: Deflection of a badminton racket during a stroke. *Journal of Sports Sciences*, 28:505–511.

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Article 2

Kwan, M., Cheng, C.-L., Tang, W.-T., and Rasmussen, J. (2010). Measurement of badminton racket deflection during a stroke. *Sports Engineering*, 12:143–153.

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Article 3

Kwan, M., Andesen, M. S., Cheng, C.-L., Tang, W.-T., and Rasmussen, J. Investigation of high-speed badminton racket kinematics by motion capture. (Submitted April 2010 to *Sports Engineering*)

6

Article 4

Kwan, M., de Zee, M., and Rasmussen J. Effects of racket swingweight on racket power in badminton. (Submitted May 2010 to *Journal of Sports Science*)

7

Racket Elasticity and Player Skill

The deformation behavior of a badminton racket during a smash stroke is studied in Article 1 (Kwan and Rasmussen, 2010). Results indicate that elite badminton players are consistently able to produce strokes where impact is timed when elastic velocity is at or near maximum. To further our study of the role of racket elasticity, the following two questions are investigated:

1. How do advanced and recreational players compare, in terms of stroke consistency and use of racket elasticity?
2. How does racket stiffness affect performance?

To address these questions, strain measurements are recorded for several strokes performed by advanced to elite and novice to recreational players, respectively referred to as Groups 1 and 2. The stroke is characterized by stroke duration, deflection speed at impact, and deflection at impact. Deflection speed at impact is an indicator of how well the player is able to exploit the racket's elasticity, and stroke consistency is assessed by standard deviations of the stroke parameters. Since measurements precisely at impact are not always reliable due to the impulsive disruption, deflection at impact is actually taken 2 ms (5 frames) before impact, and deflection speed at impact is read 4 ms (10 frames) before impact.

A few modifications to the original experimental setup in Article 1 are made. Strokes are performed in a more controlled condition, where the shuttlecock is hung from the ceiling, rather than being hit back and forth as in the previous study. By eliminating the variation in the trajectory of the incoming shuttlecock, variation in the player stroke could be isolated. In addition, a higher sampling rate of 2400 Hz is used for increased accuracy, compared to the 300 Hz used previously.

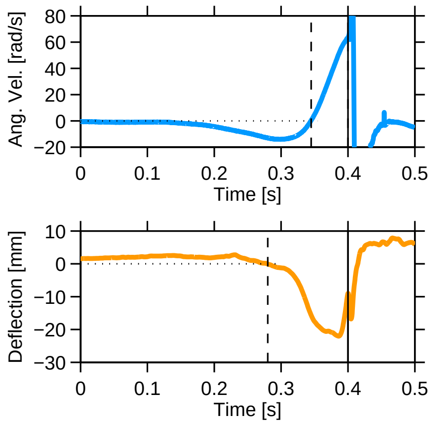

Figure 7.1: Angular velocity and racket head deflection profiles during a stroke, respectively showing forward swing duration and "foreswing" duration as defined in Article 1. Impact occurs at $t = 0.4$ s.

A new stroke duration is also defined. In Article 1, the stroke duration was measured from the time of zero strain to the time of impact. While this measurement was relatively straightforward for elite players, recreational players often lack a significant backswing which makes the time of zero strain difficult to distinguish. A new stroke duration that can be measured reliably is therefore defined as the time of peak strain to the time of impact.

It is important to note that the magnitude of this new stroke duration, which we will call peak-to-impact time, does not hold any real significant meaning for the stroke, just as the "foreswing duration" in Article 1 was defined out of convenience. True durations of the stroke phases should be derived from racket velocities, not elastic behavior. The difference between these two duration definitions can be seen in Fig. 7.1. The intended use of the peak-to-impact times measured here is for assessing the variation within each player and comparing relative differences in magnitudes, rather than computing actual magnitudes.

7.1 Strains and player skill level

In this study, a single racket (FZ Kevlar N-Power 200) is used by a group of twelve recreational (Group 2) and eight advanced (Group 1) players. The mean and standard deviation of the deflection at impact, deflection

Player Name	Group No.	Impact defl. speed [m/s]	Impact deflection [mm]	Peak-to-impact time [ms]
MKE	2	2.180 ± 0.774	6.72 ± 8.32	46.3 ± 4.7
JJA	2	1.000 ± 0.467	-2.63 ± 3.37	49.2 ± 8.1
EL	2	1.439 ± 0.069	3.01 ± 0.79	52.1 ± 1.8
JHA	2	0.936 ± 0.269	10.15 ± 2.93	59.6 ± 4.3
JR	2	0.592 ± 0.077	-6.74 ± 0.60	50.6 ± 2.5
BNI	2	1.169 ± 0.388	20.96 ± 6.80	54.1 ± 2.5
LZH	2	1.225 ± 0.178	-1.97 ± 2.38	44.2 ± 1.2
THL	2	1.773 ± 0.332	-0.60 ± 9.34	43.3 ± 6.8
HJB	2	2.309 ± 0.473	26.04 ± 10.90	50.8 ± 7.2
CGO	2	3.298 ± 0.435	16.84 ± 4.80	39.0 ± 3.2
ALH	2	1.121 ± 0.256	5.63 ± 3.42	52.3 ± 5.9
TC	2	1.016 ± 0.251	-3.13 ± 3.58	53.8 ± 4.1
KPH	1	1.892 ± 0.121	-3.81 ± 1.69	51.9 ± 2.4
JL	1	1.427 ± 0.144	-9.76 ± 1.01	39.2 ± 1.1
MN	1	0.986 ± 0.197	-8.32 ± 3.61	46.8 ± 3.4
RZ	1	0.909 ± 0.066	-2.68 ± 2.02	46.2 ± 1.6
HK	1	0.727 ± 0.060	-8.77 ± 1.35	44.8 ± 2.5
HK2	1	0.730 ± 0.072	-11.55 ± 2.87	41.8 ± 4.3
TQ	1	0.883 ± 0.097	-12.28 ± 2.20	36.9 ± 2.1
RZ2	1	0.966 ± 0.111	-15.47 ± 2.70	37.9 ± 1.2

Table 7.1: Stroke parameters of Group 1 (advanced) and Group 2 (recreational) players.

speed at impact, and stroke duration for each player are given in Table 7.1. During the forward swing, the racket reaches its maximum negative deflection and subsequently begins its positive ascent towards zero. A negative deflection at impact indicates that the racket has not yet returned to its undeformed position, while a positive impact deflection indicates the racket has continued past its original position. Using zero deflection as a reference, an impact at a negative deflection can be considered "early", while an impact at a positive deflection could be called "late".

The results indicate that all players are able to take advantage of the racket elasticity. Every recreational player strikes with an elastic velocity that contributes positively to the total racket velocity. Some recreational players even generate elastic velocities which exceed those of advanced

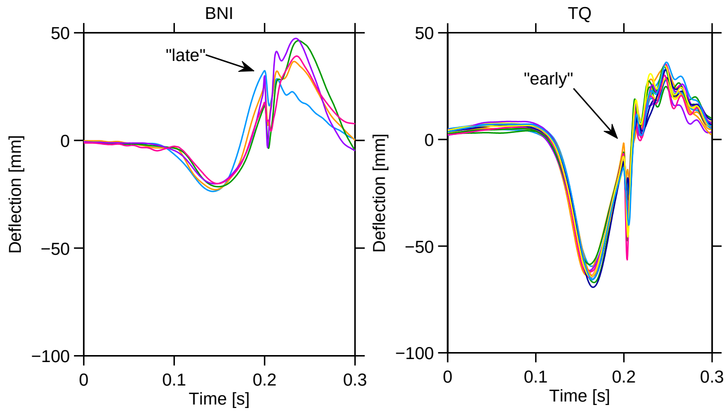

Figure 7.2: Deflection profiles for a Group 2 player (left) and a Group 1 player (right).

players. Group 1 players generate elastic velocities from 0.7 to 1.8 m/s, and players from Group 2 generate elastic velocities both higher and lower, 0.6 to 3.3 m/s. Variation in elastic velocity tends to be smaller for Group 1 than Group 2 (maximum 0.252 vs. 0.774 m/s), implying that advanced players can consistently generate their desired elastic response, whereas some recreational players are less able to consistently use the racket elasticity (see Fig. 7.3). Higher elastic impact velocities appear to lead to higher variation, as displayed by the two Group 2 players with the highest elastic speeds, CGO and HJB.

Mean impact deflections range from -6.7 to 26.2 mm for Group 2 and -15.5 to -2.7 mm for Group 1. As shown in Fig. 7.2, advanced players generally strike with a more negative deflection, or “earlier”, than recreational players, when the racket tip is still on its way to its nominal undeflected position. Recreational players often strike “later” with a decidedly positive deflection, after the racket tip has continued beyond its nominal position. Standard deviations for Group 1 players remain under 4 mm, whereas a couple Group 2 players reach standard deviations around 10 mm. Mean peak-to-impact times range from 39 to 60 ms for Group 2, and 37 to 52 ms for Group 1. Advanced players therefore tend to have slightly shorter peak-to-impact times. Standard deviations range from 1.2 to 8.1 ms for Group 2, and 1.1 to 4.3 ms for Group 1.

In summary, the standard deviations in Table 7.1 suggest that some recreational players are capable of hitting as consistently as advanced

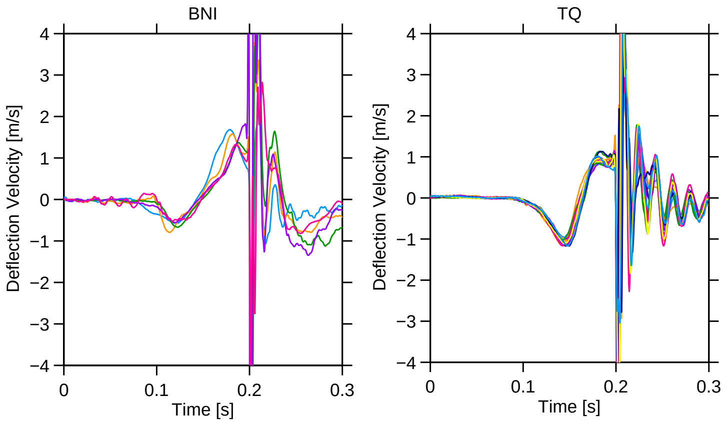

Figure 7.3: Deflection velocity profiles for a Group 2 player (left) and a Group 1 player (right).

players, while others exhibit very large variations in their stroke. Racket elasticity is beneficial to all players, and both groups of players can generate similar impact elastic velocities. However, most advanced players choose to limit their elastic velocities at impact to below 1 m/s, whereas most recreational players demonstrate velocities over and sometimes well over 1 m/s.

7.2 Strains and racket compliance

In the previous section, we discussed the relationship between stroke parameters and player skill. Here, we investigate how performance is affected by the stiffness of the racket, and whether this relationship is also dependent on player skill level. In this study, racket stiffness is characterized by the fundamental frequency of the racket, which is dependent on both mass and stiffness properties of the racket. Since the two rackets have near identical mass properties, racket compliance can be described by the racket fundamental frequency, ω_n.

Using two specially manufactured rackets on opposite ends of the elasticity spectrum, one extremely flexible (ω_n=10.0 Hz) and one extremely stiff (ω_n=18.5 Hz), strain measurements are recorded during several strokes performed by five recreational (Group 2) players and two advanced (Group 1) players. Recreational players use a non-moving target, a shuttlecock hung from the ceiling, while elites have the shuttlecock thrown to them.

Deflection at impact, deflection speed at impact, and peak-to-impact time are calculated from the strain data, shown in Tables 7.2 and 7.3.

All players generate higher deflection speeds at impact with the ultra-flexible racket (1.3-4.3 m/s) than with the ultra-stiff racket (0.5-1.3 m/s), indicating the softer rackets offer more of an additional boost to the racket head speed at impact. It also seems that Group 1 players (1.1-1.3 m/s) are better able to exploit the elasticity of the ultra-stiff racket than Group 2 (0.5-1.1 m/s). Deflection at impact is generally more negative for the softer racket (except SR), implying that the racket is still bent backward and has not yet reached its undeformed state by the time of impact. Advanced players tend to have slightly shorter peak-to-impact times with the ultra-stiff racket (20-29 ms vs. 26-29 ms), while recreational players have slightly longer peak-to-impact times with the ultra-stiff racket (39-83 ms vs. 34-77 ms).

Standard deviations in impact deflection and deflection speed are generally larger for ultra-flexible racket (except JR), but not in peak-to-impact time. Although variations in stroke timing are often greater in the ultra-stiff racket, variations in deflection and deflection speed at impact are much greater for the ultra-flexible racket. This indicates that flexible rackets are much more sensitive to small differences in stroke timing than stiffer rackets. To summarize, a softer racket leads to higher elastic velocity at impact as well as larger variation in deflection and deflection speed at impact. Greater racket compliance therefore translates to increased racket power but decreased control.

A separate study is conducted using a pair of relatively flexible and stiff rackets that are available commercially, the FZ Titanium 160 N-Forze (ω_n=14.7 Hz) and the Forza Kevlar N-Power 200 (ω_n=16.0 Hz). Similar effects of racket elasticity are observed from the results, shown in Tables 7.4 and 7.5. Slightly higher elastic velocities are measured with the softer Ti 160 racket for all players, and larger variations in impact deflection and deflection speed (except MN, BNI). Deflection at impact was more positive for the stiffer Kev 200 racket (except BNI). Peak-to-impact times are similar for both rackets, though slightly shorter with the stiffer racket for advanced players, as observed before. Differences between the two rackets are not as pronounced as those from the previous study, but the data displays similar trends nonetheless.

Player Name	Group No.	Impact defl. speed [m/s]	Impact deflection [mm]	Peak-to-impact time [ms]
JR	2	2.341 ± 0.116	-23.09 ± 2.68	44.3 ± 1.0
MSA	2	2.757 ± 0.634	-4.20 ± 5.02	47.4 ± 1.5
CDC	2	3.097 ± 0.618	-27.39 ± 6.02	40.6 ± 2.2
SR	2	1.303 ± 0.275	4.90 ± 6.51	76.6 ± 10.4
CGO	2	4.019 ± 0.309	-41.96 ± 6.64	34.3 ± 1.6
ELO	2	2.784 ± 0.577	-35.44 ± 6.94	38.8 ± 2.5
LS1	1	4.113 ± 0.706	-33.52 ± 6.91	26.4 ± 3.7
LS2	1	4.268 ± 0.504	-34.41 ± 5.78	27.6 ± 3.7
FRK	1	2.513 ± 0.248	-33.42 ± 3.79	29.3 ± 3.0

Table 7.2: Stroke parameters for smash strokes of Group 1 and Group 2 players using ultra-flexible racket.

Player Name	Group No.	Impact defl. speed [m/s]	Impact deflection [mm]	Peak-to-impact time [ms]
JR	2	0.526 ± 0.134	-1.74 ± 2.22	47.5 ± 5.2
MSA	2	0.931 ± 0.065	4.78 ± 1.98	50.9 ± 3.1
CDC	2	0.549 ± 0.160	0.26 ± 1.09	52.0 ± 3.1
SR	2	0.535 ± 0.264	3.66 ± 1.60	83.3 ± 11.0
CGO	2	1.126 ± 0.147	-2.22 ± 4.12	38.5 ± 4.5
ELO	2	0.605 ± 0.147	-6.15 ± 1.88	42.4 ± 2.4
LS1	1	1.251 ± 0.261	-7.75 ± 2.05	19.8 ± 5.1
LS2	1	1.308 ± 0.269	-10.21 ± 1.77	22.0 ± 8.0
FRK	1	1.069 ± 0.132	-8.30 ± 1.36	28.5 ± 3.3

Table 7.3: Stroke parameters for smash strokes of Group 1 and Group 2 players using ultra-stiff racket.

7.3 Comments on racket elasticity and player skill

Greater racket compliance leads to higher elastic velocity at impact, but also greater variation in the stroke. In performance terms, a softer racket offers more power (higher elastic velocities), while a stiffer racket offers more control (less stroke variation). Finding the ideal racket stiffness lies then in determining the levels of racket power and control that are optimal for a given player. This balance is likely related to player skill level.

Recreational players generally have lower racket swing speeds and strike

Player Name	Group No.	Impact defl. speed [m/s]	Impact deflection [mm]	Peak-to-impact time [ms]
MN	1	1.077 ± 0.165	-14.58 ± 2.88	47.3 ± 2.3
RZ	1	0.981 ± 0.234	-7.34 ± 3.08	48.8 ± 3.5
HK	1	1.031 ± 0.123	-11.99 ± 2.99	46.5 ± 2.5
ASK	2	1.485 ± 0.187	7.02 ± 5.77	47.9 ± 1.8
BNI	2	0.756 ± 0.111	20.09 ± 3.41	58.0 ± 3.1
LZH	2	0.776 ± 0.158	-4.75 ± 4.23	45.7 ± 1.4
MEL	2	0.959 ± 0.170	-16.94 ± 4.09	41.3 ± 1.5

Table 7.4: Stroke parameters for smash strokes of Group 1 and Group 2 players using Ti 160 racket.

Player Name	Group No.	Impact defl. speed [m/s]	Impact deflection [mm]	Peak-to-impact time [ms]
MN	1	1.009 ± 0.198	-8.72 ± 3.58	46.8 ± 3.6
RZ	1	0.909 ± 0.066	-2.68 ± 2.02	46.2 ± 1.6
HK	1	0.727 ± 0.060	-8.77 ± 1.35	44.8 ± 2.5
ASK	2	1.289 ± 0.062	10.38 ± 1.97	47.4 ± 1.5
BNI	2	0.679 ± 0.161	15.96 ± 5.34	57.8 ± 3.1
LZH	2	0.613 ± 0.130	-1.32 ± 3.68	45.8 ± 1.9
MEL	2	0.782 ± 0.121	-11.71 ± 1.13	39.1 ± 1.3

Table 7.5: Stroke parameters for smash strokes of Group 1 and Group 2 players using Kev 200 racket.

less consistently than advanced players. These players could benefit more from a softer racket, using the extra elastic velocity to boost the racket impact velocity while relatively oblivious of the decreased level of control, if their stroke consistency is not particularly high. Advanced players may prefer stiffer rackets over softer rackets, since they offer more control. With their level of skill, they are able generate high racket speeds, so maximizing the relatively small contribution from elastic velocity may not be the appropriate optimization. Finding the racket stiffness that maximizes control is perhaps more appropriate.

Generally speaking, advanced players do prefer stiffer rackets, whereas more flexible rackets are recommended for novice players. Since racket power is mostly generated by the speed of the racket head rather than from the elastic velocity, racket stiffness could be considered more relevant to control than power. Control should therefore be the primary optimization criteria for finding the ideal racket compliance, with power as a secondary goal. Using stroke variation as an indicator of player skill level, finding the relationship between stroke variation and racket stiffness for an individual player may be the key to identifying the ideal racket stiffness.

Looking at Tables 7.2-7.5, we can get an impression of the stroke variation-racket stiffness relationship by plotting the span of standard deviations for the advanced and recreational players. Minimum and maximum standard deviations of the impact deflection velocity and impact deflection for the ultra-flexible (10.0 Hz), Ti 160 (14.7 Hz), Kev 200 (16.0 Hz), and ultra-stiff (18.5 Hz) rackets are plotted in Fig. 7.4 and Fig. 7.5.

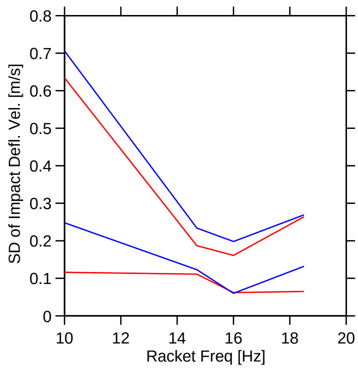

Figure 7.4: Minimum and maximum sd of impact deflection velocity vs. racket frequency of recreational (red) and advanced (blue) players.

Stroke variation decreases rapidly with increasing racket stiffness (frequency) and then appears to level off. A clearer trend could be seen with more stroke trials and with all four rackets used the same players. However, with the limited data at hand, we can make some further speculations. Since we know that advanced players prefer stiffer rackets than recreational players, the relationship between stroke variation and racket stiffness may look something like Fig. 7.6, for a novice player and an elite player.

The optimal stiffness could be identified as where stroke variation is minimal (maximizing control), but as flexible as possible (maximizing power). This hypothesized relationship between stroke variation, racket stiffness, and player skill level needs to be verified through a more thorough study involving multiple test rackets and larger player groups.

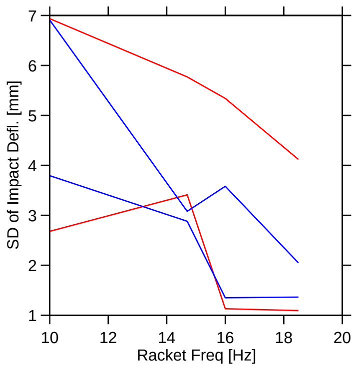

Figure 7.5: Minimum and maximum sd of impact deflection vs. racket frequency of recreational (red) and advanced (blue) players.

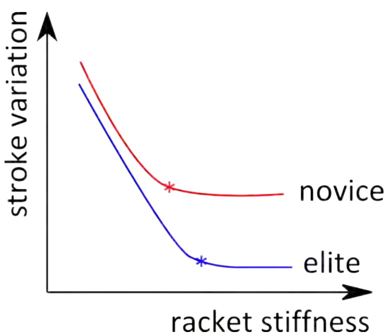

Figure 7.6: Hypothesized stroke variation vs. racket stiffness relationship for an elite and novice player. Optimal stiffness indicated by *.

8

Discussion

The success of a racket design is measured by how well the racket performs. The performance of a racket is most often described in terms of power and control. These concepts are rather abstract, in that they lack any clear mathematical definition, and must somehow be interpreted into measurable quantities for an analytical approach to racket design to be possible. In this thesis, racket power is interpreted as the momentum transferred from the racket to the shuttlecock at impact, and control is related to the variation of the stroke at impact. Racket momentum is influenced by the mass distribution and speed of the racket; standard deviations of elastic deflection and elastic velocity at impact are used to measure stroke variation.

The analytical approach to badminton racket design was centered on the study of stroke dynamics, and how dynamic response is affected by racket properties. The analysis focused mainly on two aspects: (1) effects of racket compliance on the deformation behavior, and (2) effects of racket swingweight on swing speed. These effects and their relevance to racket performance are the building blocks for an optimization scheme to determine what the ideal racket properties are for a given desired response. The desired performance characteristics can vary from player to player, but some generalizations relating ideal racket properties and player skill level can be made from observing differences in stroke dynamics between advanced and recreational players.

8.1 Effects of racket compliance on deformation behavior

The deformation behavior of the racket during a stroke can be detected using motion capture techniques, but strain gages provide more reliable measurements, as discussed in Article 2 (Kwan et al., 2010). However, strains must be scaled in order to determine deflection magnitudes. This scaling factor can be computed with a beam model, which requires measured parameters from the racket.

Since they are attached directly on the racket, strain gages have a higher sensitivity to the racket motion than motion capture, as well as a higher sampling rate capability. The deformation profile over a stroke is therefore useful in assessing performance aspects like precision and control. Deflections, however, do not correlate with racket speeds, so the elastic response is of limited use for evaluating total racket power. Only the extra power boost provided by elastic velocity, generally around 5% of the racket head velocity, can be evaluated.

Since impact is the most crucial moment of the stroke, we are most interested in the elastic deflection and elastic velocity at this moment. Over the stroke, a clear negative peak in the deflection can be seen during the forward swing, which is sometimes preceded a distinct positive peak, depending on the intensity of the player's backswing. A negative deflection at impact indicates that the racket has not yet returned to its undeformed state ("early" impact), whereas a positive deflection indicates that impact occurred after the racket tip continued past its undeflected position ("late" impact). A positive elastic velocity indicates a positive contribution to the total racket velocity, while a negative elastic velocity implies the opposite, where the racket head is moving away from the shuttlecock at impact.

Greater elasticity in the racket generally leads to a greater elastic speed at impact and an "earlier" impact, i.e. a more negative impact deflection. No clear effects of racket elasticity on peak-to-impact times were observed. Peak-to-impact times were slightly longer with the more flexible rackets in advanced players, but slightly shorter in recreational players. A softer racket also led to greater variation in both deflection and deflection speed at impact, i.e. less stroke consistency. From a performance perspective, these observations indicate that flexible rackets provide more power but less control.

8.2 Effects of racket swingweight on swing speed

Position data collected using motion capture were used to determine the kinematics of the racket during a stroke, including the velocity of the racket head center. Previous studies in tennis and baseball indicated that swing speed was nearly independent of total mass, but strongly correlated with swingweight. The relationship between swing speed V and swingweight I_o

was expressed as a power law of the form, $V = C/I_o^n$, with typical values of $n = 0.25$. Although the swingweights of badminton rackets are considerably lighter than those of tennis rackets and baseball bats, the same power law seems to apply, and similar values of n were found.

From Article 4, a general decrease in swing speed was observed with rackets of heavier swingweight. For some players, a very light swingweight can also lead to a slight decrease in impact speed, compared to the swing speed with a typical racket swingweight. One possible explanation is the lack of proprioceptive feedback that a very lightweight object provides to the player, leading to a less than optimal coordination pattern for generating maximum speed. This phenomenon has been observed in one other study (Mitchell et al., 2000), but it appears the subject has not been investigated thoroughly in the literature. Nevertheless, the main conclusion is that there is a limit to how quickly a racket of very light swingweight can be swung.

Racket swing speed, however, is not exactly synonymous with racket power. Shuttlecock speed would be a more appropriate measure of racket power, defined here as the momentum transferred from the racket to the shuttlecock. The momentum of the racket is dependent on both racket speed and swingweight. Although a heavier racket swingweight is swung slower, its larger swingweight could theoretically impart greater momentum to the shuttlecock than a lighter swingweight racket with a higher racket speed. Results from Article 4 indicate that for the range of swingweights tested, advanced players were able to generate similar shuttlecock speeds using rackets of both light and heavy swingweights. However, this could mostly be attributed to a shift in the impact location rather than the difference in racket swingweight. Using an impact model, we can fix the location of impact along the racket head, and the results show that shuttlecock speed actually decreases with increasing swingweight, just as racket speed does. A very head-heavy racket therefore does not provide more power, but it is difficult to identify a single swingweight that maximizes racket power. For advanced players, similar shuttlecock speeds are found over a rather broad range of the lighter racket swingweights. The effects of racket swingweight on control of the racket, rather than on racket power, may therefore be more appropriate for finding an optimal swingweight for some players. For other players, there may exist a clearer optimal swingweight based on power.

To quantify swingweight effects on racket control, we can calculate standard deviations of the impact racket head speed and the forward swing duration to determine stroke variation. Using data from Article 4, results are given

	Impact head speed SD [m/s]				Fwd swing duration SD [mm]			
Player	R1	R2	R3	R4	R1	R2	R3	R4
A	1.987	2.063	2.444	2.302	2.55	2.76	3.55	3.17
B	2.384	1.572	1.599	1.644	5.35	3.54	3.17	4.84
C	1.493	0.790	1.711	1.299	2.16	2.80	3.78	2.95

Table 8.1: Standard deviations of impact racket head speed and forward swing duration for players A-C.

in Table 8.1 for the three advanced players. Looking at the head velocities, the heavier swingweight rackets R3 and R4 have higher variations than R2 for all players. For players B and C, there is also greater variation in the lightest swingweight racket R1, suggesting a reduced level of control for a racket that could be considered "too light". Similar trends can be seen in the forward swing durations. For players A and C, we see higher variation in R3 and R4, compared to R1 and R2. For player B, variation is highest for rackets R1 and R4, while variations of R2 and R3 are similar. From these results, we might suggest a racket similar to R1 for player A, R2 or R3 for player B, and R2 for player C.

8.3 Observations on player skill level

Individual players have individual racket preferences, indicating that the desired performance characteristics of a racket can vary from player to player. However, finding the link between the player's ability and performance needs is not straightforward. Studying differences between advanced and recreational players could provide clues about how to quantify this relationship.

From strain measurements, different trends in elastic behavior can be observed between the two groups. Deflections at impact are decidedly negative for all advanced players, meaning they strike before the racket has returned to its undeflected state, or "early". Impact deflections for recreational players span the entire range from "early" to "late". The reasons behind the consistent "early" impact of the advanced players remain unknown. A "late" impact does not imply a lower deflection speed, and an "early" impact does not guarantee a higher deflection speed. The driving factor for this "early" impact trend must therefore be something other than maximizing deflection speed at impact. This may be further evidenced by the fact that advanced players do not always generate higher impact deflection speeds than recreational players. Some recreational players have lower deflection speeds, while others are capable of generating much higher speeds. Generally, peak-to-impact times are shorter for advanced players, especially with stiffer rackets, than for most recreational players, but not all. A recreational player may have a shorter peak-to-impact time than an advanced player, but this can likely be attributed to a shorter swing length rather than higher swing speeds. Advanced players generally exhibited lower stroke variation than recreational players, though not consistently. A larger number of trial strokes may be necessary to improve accuracy of the standard deviation calculation. We would expect that over a larger pool of players and a greater number of trials, advanced players would be more consistent, exhibiting less variation in elastic deflection and elastic velocity than novices.

Differences in racket kinematics during the stroke can be seen between recreational and advanced players, as discussed in Article 3. Racket motion is described by angular kinematics of the racket and linear kinematics of the handle base. Advanced players are able to generate remarkably high angular velocities and accelerations, while linear velocities and accelerations are roughly comparable to those of recreational players. Some recreational players even generate higher linear velocities than advanced players. However, the resultant racket head velocities generated by advanced players are significantly higher than those generated by recreational players. Racket head velocity can be decomposed into translational and rotational components, calculated from the linear velocity of the handle base and the angular velocity of the racket, respectively. The rotational component is considerably larger than the translational component, for all players. However, the rotational contribution is higher for advanced players than for recreational players, since advanced players are able to generate higher angular velocities than recreational players. Differences in technique and coordination of the arm segments between the two groups are further evidenced by dissimilarities in the overall shape and timing of kinematic profiles. A strong rotational component appears to be most effective in generating high racket head speeds.

Effects of swingweight on swing speed appear relatively low for advanced players, but recreational players may be more sensitive to changes in racket swingweight. A larger test population is necessary to make more conclusive observations. The advanced players also demonstrate greater stroke consistency, based on the scatter of impact locations on the racket head over several trials. A much more focused group of impact locations can be seen with the advanced players than the recreational players.

8.4 Implications for racket design and performance

Rackets offer different levels of control and power. Players possess different levels of skill and strength. To match the player with the ideal racket, we need to determine the levels of power and control that are best suited to the player. Player skill and strength cannot be directly quantified easily, but is perhaps better represented through performance factors, by measuring characteristics such as swing speed and variation of the stroke.

8.4.1 Optimization of racket properties

Optimization of racket swingweight can be approached from a power-based method, where the player's swing speed is measured over a range of different racket swingweights. For advanced players, we saw that swing speed remains relatively unchanged over a broad range of swingweights. This indicates that a power-based optimization may not be entirely suitable for finding the optimal swingweight. A control-based optimization may therefore be more appropriate for finding an ideal swingweight for an advanced player, where stroke variation rather than swing speed is measured. Recreational players may be more sensitive to changes in swingweight, in which case the original power-based optimization would still be suitable.

The optimal racket stiffness can be assessed by a control-based method, where effects of stiffness on stroke variation are determined. Variation generally decreases as racket stiffness increases, until a limit is reached where variation decreases no further. This "base" variation can be viewed as a measure of the consistency in the coordination of the player's stroke, dependent on the player's level of skill. This concept is illustrated in Fig. 7.6. We would expect that this base variation is higher for a recreational player, and therefore reached at a lower stiffness. From this point, a higher stiffness does not offer a greater degree of control to the player, only a decrease in elastic velocity. This point can therefore be considered the optimal stiffness, where control is maximized first (minimum variation), then power (maximum elasticity). For the elite player, the base variation would be lower and presumably reached at a higher stiffness. This could explain why advanced players prefer stiffer rackets with more control and less power, while recreational players prefer softer rackets that offer less

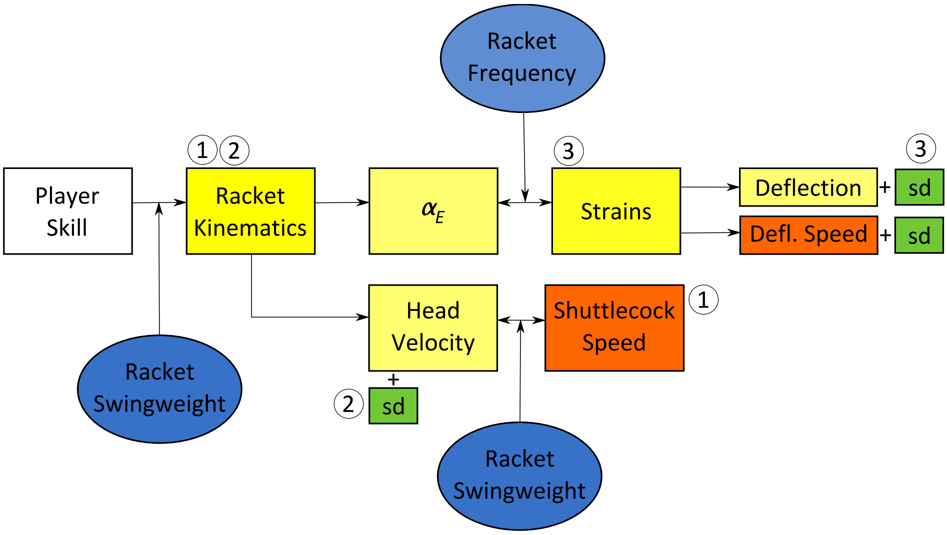

Figure 8.1: Model of relationships between player, racket, and performance. Racket properties (blue) and stroke parameters related to power (orange) and control (green) are shown. Processes for determining (1) power-based optimal swingweight, (2) control-based optimal swingweight, and (3) control-based optimal frequency labelled in circled numbers.

control but more power. However, more data is needed to confirm this theory.

The relationships between player, racket, and performance that are central to the optimization of racket swingweight and stiffness are represented in Fig. 8.1. Racket kinematics are a product of player skill and racket swingweight. From the kinematics, we obtain racket head velocities. Shuttlecock speeds, dependent on racket head velocities and swingweight, are then used to find a power-based optimal swingweight, labelled as (1). The control-based optimal swingweight, labelled (2), can be determined from standard deviations of the racket head speeds. Accelerations of the racket induce inertial loads, represented by α_E, which cause the racket to deform. The effects of racket compliance can be measured using strain gages. The control-based optimal racket frequency, labelled (3), is then determined from standard deviations of the elastic deflection and velocity at impact.

8.4.2 Racket fitting model

While generalizations about racket preferences of advanced vs. recreational players can be made, individual style and technique can vary considerably between players. Finding the optimal racket properties therefore requires measurements from the individual player. The most accurate method for "fitting" a racket to a player would involve several measurements with many different rackets to generate accurate relationships between performance factors and racket properties, e.g. power vs. swingweight and control vs. stiffness curves. However, this would be a rather time-consuming, labor-intensive process. It would be ideal to have a model capable of predicting how a new racket would affect the stroke dynamics for a given player, based on just a few measured strokes.

The relationship between swing speed and racket swingweight for a given player is very much dependent on player skill and strength. Modeling this relationship could be possible with an extensive human model, but incorporating the appropriate skill and strength parameters would be challenging. A more accurate method would be to simply measure this relationship experimentally, using a few rackets of very different swingweights. The power law could then be applied to curve fit a number of data points. Similarly, the relationship between stroke variation and racket swingweight should be measured, and subsequently curve-fitted. Although aspects of player skill and strength must be determined experimentally, certain physical phenomena pertaining to the racket can be modeled relatively reliably. Swing speed can be transformed into shuttlecock speed quite reliably using a momentum model, given the swingweight of the racket (and impact location). The shuttlecock speed vs. swingweight curve can then provide the power-based optimum for racket swingweight.

Modeling the effects of racket compliance on stroke variation may be more feasible. From strain measurements of a given racket, we can measure the variation in impact deflection and deflection speed for a given player. Using a beam model, it is possible to extract a loading function α_E which the racket is subjected to over the stroke, caused by accelerations generated by the player. Assuming the player does not adapt his stroke to the racket, the same loading function can be applied to a racket of a different stiffness in order to predict the deflection profile of this new racket if it were swung in the exact same way as the original. New stroke variation measurements can then be taken to construct the variation vs. stiffness curve. With the current beam model, however, the loading function is not a purely based on kinematics, but also dependent on a parameter of the racket, the beam length. Unfortunately, the beam length cannot be conveniently extracted from α_E, so the model is only valid when comparing rackets with the same beam length. It may be possible to predict the deflection behavior of a new racket by developing a more complex beam model, but with the current model, it seems the only solution is to determine the effects of racket stiffness on stroke variation experimentally.

In summary, an individualized fit of a racket can be determined from the following optimization schemes (also labeled in Fig. 8.1):

1. *Ideal swingweight (power-based).* Measure racket speeds for varying racket swingweights. Use the momentum model to predict shuttlecock speeds from racket speeds. Plot shuttlecock speed vs. racket swingweight. Find the maximum shuttlecock speed.

2. *Ideal swingweight (control-based).* Measure racket strains for varying swingweight. Plot variation of stroke parameters vs. swingweight. Find the minimum variation.

3. *Ideal stiffness.* Measure racket strains for varying racket frequencies. Plot variation of stroke parameters vs. racket frequency. Find the minimum variation while minimizing racket frequency (maximizing racket compliance).

Racket compliance is characterized by the fundamental frequency of the racket, rather than the flexural modulus, since racket frequency is a racket property that can be measured accurately as well as a required input to the beam model. With the optimal racket swingweight and fundamental frequency, the optimal racket compliance can be calculated, if needed. In the optimization of the racket stiffness, control is prioritized over power. While this appears to be valid for advanced players, whether it is the correct assumption for recreational players remains to be determined.

8.5 Future Work

The science of badminton racket design is quite new, and surprisingly little is known about why the rackets are made the way they are. The goal of this thesis was to methodically investigate the underlying physics of the game and begin establishing a clear foundation for the ideal racket design, or more accurately, the ideal racket-player pairing.

Some of the issues addressed in this work require further investigation, including the dependence of the stroke variation-racket stiffness relationship on player skill. This can be accomplished by recording strain measurements during the stroke for several trials performed by players of different skill levels using a selection of rackets with different stiffnesses. Establishing this relationship would help to better assess the aspects of racket control in the racket-player interaction, and ultimately to determine the appropriate racket parameters. To further verify the results, it may be possible to develop a model that can predict this relationship. The beam model presented here was not capable of doing so. Using the deflection behavior (or perhaps the racket kinematics) during the stroke for a known racket stiffness (or frequency), it seems feasible that a model could generate the deflection behavior of a new racket stiffness or frequency input. Development of a working model would further our understanding of the physical phenomena at work, and would facilitate the racket fitting process as well.

The main purpose of establishing the relationship between stroke variation and racket stiffness is based on the proposed idea that the optimal stiffness maximizes control. However, it is uncertain whether minimum variation is the actual driving factor of optimal stiffness. Another theory could be based on the deflection at impact observed in advanced players. Impact deflections for advanced players are consistently "early" or near-zero, but the reasons for this pattern remain a mystery, since an "early" impact deflection does not necessarily coincide with the maximum elastic speed. Finding the true explanation could help to better clarify the criteria for optimal racket stiffness as well.

It would also be interesting to further explore the effects of ultra-light swingweight rackets. In some players, we observed a slight decrease in racket head speed and an increase in stroke variation, which is possibly explained by the lack of proprioceptive feedback provided by such a light swingweight. The information could be helpful in finding the lower limit of appropriate racket swingweights for a given player. The challenge in performing such a study is the lack of ultra-light swingweight rackets available. One possible solution may be to use a racket without strings, or to simply test generic long, thin rods as done in Cross and Bower (2006).

Another possible research direction is to address the limitations in the measurement systems available. The proposed motion sensor device could prove to be a new method of data collection that is both faster and more reliable. By measuring velocities and accelerations directly, this improves the accuracy and significantly reduces processing time, especially compared to video methods. Preliminary tests indicate good agreement of the measured racket kinematics from the motion sensor with motion capture data. However, further tests are necessary to determine whether the device is sensitive enough to evaluate stroke variation, thus far only possible with strain measurements. If the capability of the device in proven, it could open many new doors for possible experiments. Being no longer restricted to a motion capture laboratory or tethered to a heavy measurement unit, the portability of the small device could take experiments on to the court, possibly even in a game situation. Possible effects of different playing conditions, between a laboratory setting and a more realistic situation, have yet to be investigated.

Following in the footsteps of previous studies in tennis, a number of the same questions can be applied to badminton, including an analysis of player sensitivity to racket swingweight and stiffness, measurements of the apparent coefficient of restitution across the racket head, and the independence swing speed to racket mass. A more complete overview of racket design could also require inclusion of performance factors other than power and control, an investigation into different types of strokes other than the smash, as well as a deeper look into the impact mechanics between the strings and the shuttlecock, e.g. effects of string tension, shuttlecock deformation and elasticity, dwell time, and sweet spots.

8.6 Concluding remarks

The work presented in this thesis provides the first glimpse into the dynamics and design aspects of the badminton racket specifically, upon which the optimization of the racket properties can be developed to make the world's best racket for the player.

A

Racket Aerodynamics

The aerodynamics of the racket are improved by minimizing drag. Since drag forces vary along the racket, resistance due to drag may be more conveniently quantified by a torque. The player must generate a torque that both accelerates the racket and overcomes drag forces during the stroke. To put the drag resistance into perspective, these two components of the total torque can be approximated by constructing a model of the racket's profile.

The racket model is composed of the handle, shaft, frame, and strings. The handle is represented by a rectangular cross-section with a drag coefficient, $C_d = 1.8$. The shaft and strings are represented by a circular cross-section with a drag coefficient, $C_d = 1.2$. The frame is represented by an elliptic cross-section with a drag coefficient, $C_d = 0.6$. The drag force F_d is given by:

$$F_d = \frac{1}{2} C_d \rho V^2 A \tag{A.1}$$

where density of air $\rho = 1.22$ kg/m^3, V is the normal velocity of the racket, and A is the frontal area of the racket. Since the velocity varies along the racket, the torque required to overcome the drag forces, which we will refer to as drag torque, can be calculated. The drag torque for each racket component is given by

$$\tau_d = \int_{x_1}^{x_2} \frac{1}{2} C_d \rho V(x)^2 W \; dx \tag{A.2}$$

where x_1 and x_2 are the start and end locations of the racket component along the racket, $V(x)$ is the normal velocity of a point at a location x along the racket, and W is the frontal width or diameter of the racket component. From racket kinematics, we have the linear velocities of the handle base, v, and the angular velocities of the racket, ω, in the local body-fixed coordinate system of the racket, xyz. We can then calculate

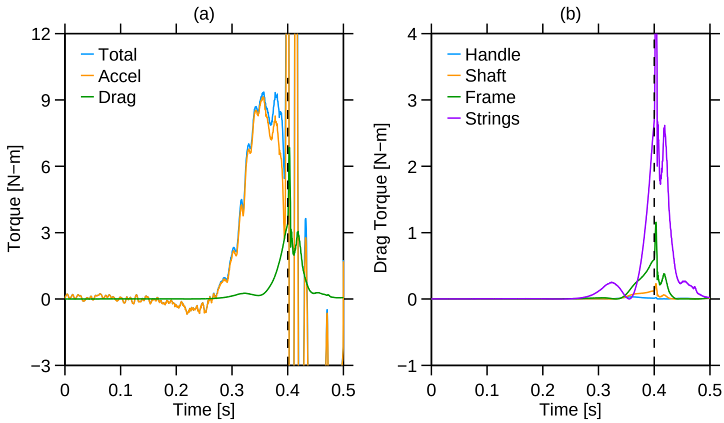

Figure A.1: (a) Total torque composed of torque required to accelerate the racket and to overcome drag. (b) Contributions of handle, shaft, frame, and strings to the drag torque.

the normal velocity, $V(x) = v_z - x\omega_y$. The total drag torque is the sum of all the drag torques of the racket components (handle, shaft, frame, and strings).

The torque required to accelerate the racket, referred to as acceleration torque, can also be calculated from racket kinematics.

$$\tau_a = I_o \alpha_N \tag{A.3}$$

where I_o is the moment of inertia about the handle base, and $\alpha_N = -\dot{\omega}_y + \omega_x\omega_z$ is the angular acceleration normal to the racket face.

Fig. A.1a shows that the acceleration torque is much larger than the drag torque. It may therefore be more useful to investigate ways to reduce the required acceleration torque, related to racket mass properties.

Focusing on the drag torque, drag contributions of each of the racket components can be seen in Fig. A.1b. The strings and frame contribute most to the drag torque. A slimmer shaft therefore provides little improvement in racket aerodynamics. Thinner strings or a more aerodynamic profile of the racket frame may be more effective in reducing drag.

B

Racket Motion Sensor

The racket motion sensor consists of a triaxial accelerometer and a triaxial gyroscope placed at the base of the racket handle. From the acceleration and angular velocity data provided by the two sensors, we can determine the velocity profile of the racket head during the stroke, and ultimately the impact velocity of the racket head.

B.1 Rigid Body Motion

We are looking for the loads on the racket due to accelerations. The motion of a point P on a rotating body with local coordinate system ijk in a global coordinate system XYZ can be described by:

$$\mathbf{r}_P = \mathbf{r}_H + \mathbf{A}\mathbf{r} \tag{B.1}$$

$$\dot{\mathbf{r}}_P = \dot{\mathbf{r}}_H + \mathbf{A}\omega \times \mathbf{r} \tag{B.2}$$

$$\ddot{\mathbf{r}}_P = \ddot{\mathbf{r}}_H + \mathbf{A}\dot{\omega} \times \mathbf{r} + \mathbf{A}\omega \times \omega \times \mathbf{r} \tag{B.3}$$

where H is the origin of the local coordinate system of the racket located at the handle base, $\mathbf{r}$ is the vector from point H to point P in local coordinates, and $\mathbf{A}$ is the rotation matrix from the local to global coordinate system.

The vectors above can be expressed in local coordinates:

$$\mathbf{A}^T\mathbf{r}_P = \mathbf{A}^T\mathbf{r}_H + \mathbf{r} \tag{B.4}$$

$$\mathbf{A}^T\dot{\mathbf{r}}_P = \mathbf{A}^T\dot{\mathbf{r}}_H + \omega \times \mathbf{r} \tag{B.5}$$

$$\mathbf{A}^T\ddot{\mathbf{r}}_P = \mathbf{A}^T\ddot{\mathbf{r}}_H + \dot{\omega} \times \mathbf{r} + \omega \times \omega \times \mathbf{r} \tag{B.6}$$

To simplify notation:

$$\mathbf{s}_P = \mathbf{A}^T\mathbf{r}_P \tag{B.7}$$

$$\mathbf{v}_P = \mathbf{A}^T\dot{\mathbf{r}}_P \tag{B.8}$$

$$\mathbf{a}_P = \mathbf{A}^T\ddot{\mathbf{r}}_P \tag{B.9}$$

B.2 Accelerometers

The acceleration of point P measured by the accelerometer is given by

$$\mathbf{a}_P^* = \mathbf{a}_P + \mathbf{A}^T\mathbf{g} \tag{B.10}$$

where $\mathbf{a}_P$ is the acceleration of point P and $\mathbf{g}$ is the gravity vector. Since we are only interested in the kinematics of the racket, we would like to remove the gravity term. In order to do so, we need to calculate the rotation matrix $\mathbf{A}$ as a function of time.

Using Euler parameters, the orientation of the racket is represented by $\mathbf{p} = [e_0\ e_1\ e_2\ e_3]$. The time derivative $\dot{\mathbf{p}}$ is given by (Nikravesh, 1988):

$$\begin{bmatrix} \dot{e_0} \\ \dot{e_1} \\ \dot{e_2} \\ \dot{e_3} \end{bmatrix} = \frac{1}{2}\begin{bmatrix} 0 & -\omega_x & -\omega_y & -\omega_z \\ \omega_x & 0 & \omega_z & -\omega_y \\ \omega_y & -\omega_z & 0 & \omega_x \\ \omega_z & \omega_y & -\omega_x & 0 \end{bmatrix}\begin{bmatrix} e_0 \\ e_1 \\ e_2 \\ e_3 \end{bmatrix}, \quad \mathbf{p}(0) = \mathbf{p}_0 \tag{B.11}$$

From this differential equation, the orientation vector $\mathbf{p} = [e_0\ e_1\ e_2\ e_3]$ can be determined as a function of time. In order to solve this equation, we first need to determine the initial conditions. Initial orientation of the racket $\mathbf{p}_0$ can be determined from the accelerometers, provided that racket begins at rest (the angular velocities are zero).

B.2.1 Finding initial orientation

Let gravity $\mathbf{g} = [0\ 0\ -9.81]$ in the global coordinate system XYZ. If we assume the racket is rotated first by the angle ϕ about the y-axis, followed by angle θ about the x-axis, the initial rotation matrix $\mathbf{A}_0$ is:

$$\phi = \tan^{-1}\left(\frac{a_z}{a_x}\right) \tag{B.12}$$

$$\theta = \sin^{-1}\left(\frac{a_y}{9.81}\right) \tag{B.13}$$

$$\mathbf{R}(\phi) = \begin{bmatrix} \cos\phi & 0 & -\sin\phi \\ 0 & 1 & 0 \\ \sin\phi & 0 & \cos\phi \end{bmatrix} \tag{B.14}$$

$$\mathbf{R}(\theta) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{bmatrix} \tag{B.15}$$

$$\mathbf{A}_0 = \mathbf{R}(\theta)\mathbf{R}(\phi) \tag{B.16}$$

where $\mathbf{a}_0^* = [a_x\ a_y\ a_z]$ is the initial accelerometer reading.

To avoid potential problems with Euler angles, the initial rotation matrix $\mathbf{A}_0$ can be expressed explicitly in terms of the components of the accelerometer measurement $\mathbf{a}_0^* = [a_x\ a_y\ a_z]$.

$$\mathbf{A}_0 = \begin{bmatrix} a_z/h & 0 & -a_x/h \\ -a_x a_y/gh & h/g & -a_y a_z/gh \\ a_x/g & a_y/g & a_z/g \end{bmatrix} \tag{B.17}$$

where $h^2 = a_x^2 + a_z^2$. The initial orientation $\mathbf{p}_0$ can then be determined from $\mathbf{A}_0$. A rotation matrix $\mathbf{A}$ can be represented by a set of Euler parameters $\mathbf{p} = [e_0\ e_1\ e_2\ e_3]$ (Nikravesh, 1988):

$$e_0 = \pm\frac{1}{2}\sqrt{1 + A_{11} + A_{22} + A_{33}} \tag{B.18}$$

$$e_1 = (A_{32} - A_{23})/4e_0 \tag{B.19}$$

$$e_2 = (A_{13} - A_{31})/4e_0 \tag{B.20}$$

$$e_3 = (A_{21} - A_{12})/4e_0 \tag{B.21}$$

B.2.2 Finding orientation over time

Solving the differential equation in Eq. B.11 with the initial condition $\mathbf{p}_0$ gives the orientation $\mathbf{p}(t) = [e_0\ e_1\ e_2\ e_3](t)$ during the stroke. The rotation matrix $\mathbf{A}$ can be determined from Euler parameters by (Nikravesh, 1988):

$$\mathbf{A} = 2\begin{bmatrix} e_0^2 + e_1^2 - \frac{1}{2} & e_1e_2 - e_0e_3 & e_1e_3 + e_0e_2 \\ e_1e_2 + e_0e_3 & e_0^2 + e_2^2 - \frac{1}{2} & e_2e_3 - e_0e_1 \\ e_1e_3 - e_0e_2 & e_2e_3 + e_0e_1 & e_0^2 + e_3^2 - \frac{1}{2} \end{bmatrix} \tag{B.22}$$

From Eq. B.22, the rotation matrix $\mathbf{A}(t)$ over the stroke can be calculated. Using accelometer measurements $\mathbf{a}_0$, the acceleration in local coordinates can now be determined:

$$\mathbf{a}_P = \mathbf{a}_P^* - \mathbf{A}^T\mathbf{g} \tag{B.23}$$

B.3 Racket head velocity

Since we are most interested in the impact velocity, we would like to have the velocity profile of the racket head. But the rigid body velocity (Eq. B.5) must be calculated carefully, since $\mathbf{v}_P \neq \int \mathbf{a}_P \; dt$!

$$\frac{d}{dt}\mathbf{v}_H \quad = \quad \mathbf{a}_H - \omega \times \mathbf{v}_H \tag{B.24}$$

The differential equation (Eq. B.24) must be solved to find the linear velocity of the racket base, $\mathbf{v}_H$. The accelerometer measurements, together with the body-fixed angular velocities ω given by the gyroscopes, can be used to determine $\mathbf{a}_H$ using Eqs. B.23 and B.6. Assuming the racket is initially at rest, the velocity profile of the racket head $\mathbf{v}_P$ is found from Eqs. B.24 and B.5.

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