Aerodynamics of a Badminton Shuttlecock

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ABSTRACT

Experiments were conducted to determine the drag coefficient of a badminton shuttlecock. Two types of testing were conducted: wind tunnel tests of a full-scale model, and drop tests using a high-accuracy radar gun. The drag coefficients calculated from these measurements were then compared to the limited data available in the literature. The range of drag coefficients measured was from 0.55 to 0.65.

I. INTRODUCTION

Badminton was invented in the mid 19th century, and originally used a feathered shuttlecock. The synthetic shuttlecock was invented in the 1950s. The plastic synthetic shuttlecock skirts are made by injection molding, and are generally more durable and cheaper than feather shuttlecocks (Cooke, 2002). A feather shuttlecock is generally asymmetric, with its weight split almost evenly between a cork base and the feathers in the skirt (Morgan). The feathers are from the right side wing of a goose. Official shuttlecocks should weigh between 4.74 and 5.50 grams. Feather shuttlecocks have 16 feathers and weigh around 5 grams. Shuttlecocks are unique, in that most other sports balls are symmetric (soccer, cricket, squash, table tennis, tennis, basketball, volleyball, etc.) The only previous studies on the aerodynamics of shuttlecocks reported in the literature were all performed by Cooke (1996, 1999, 2002), and there is also the work of Morgan (1996), which focused on the design of an automated shuttlecock serving machine.

The length of an official badminton court is 13.4 m. Note that the playing conditions (altitude, temperature, humidity) affect the air density, which it turn will affect how far the shuttlecock can travel. Top players can hit shuttlecocks hard enough to given them an initial velocity of over 150 mph (70 m/s) in professional competition. After being hit by a racket, a shuttlecock usually completely changes direction after traveling a distance of 20 to 80 cm. A shuttlecock looses about half of the initial velocity imparted to it by the racket by the time it completes its turnaround. Shuttlecocks do no follow the nearly parabolic trajectories of other sports balls, but because of their low mass and high drag, follow more of a parachute trajectory (Morgan).

The non-dimensional drag coefficient is calculated according to Equation 1, and the Reynolds number calculated according to Equation 2.

$$F_D = \frac{1}{2}(C_D \rho A V^2) \quad (1)$$

$$Re = \frac{\rho V d}{\mu} \quad (2)$$

Dimensional analysis can be used to show that the drag coefficient, C_D, is a function solely of the Reynolds number, Re, for objects traveling much slower than the speed of sound.

The shuttlecocks used by Cooke were of skirt diameter 65.5 - 66 mm, with a nose diameter of 26.5 mm and length of shuttlecocks typically between 81 and 82 mm. She studied both synthetic and feather shuttlecocks. Three different types of synthetic shuttlecock design - termed ProCork, Tournament, and Championship, were analyzed in the 2002 study.

Cooke (1996) tested up to 45 m/s, or a Reynolds number of 200,000, and speeds as low as 3 m/s (Re = 13,000). She found drag coefficients for synthetic shuttlecocks were usually lower than those for feather shuttlecocks, which she attributed to skirt deformation of the synthetic shuttlecocks, while the stiffer feather shuttlecocks did not deform as much. There is greater porosity and airflow through the skirt of a synthetic shuttlecock than a feather one (Cooke, 1999). Cooke (1999) found the drag coefficients of the shuttlecocks to be nearly constant and not change with Reynolds number, for Reynolds number from 13,000 to 200,000, which is also consistent with trends seen for spheres and other bluff objects that experience vortex shedding. The measured drag coefficients were between 0.4 and 0.5, with the feather shuttlecocks having the higher drag coefficients (average of 0.48). Cooke also states that shuttlecocks can be considered as bluff bodies, in which most of the drag is base drag. For bluff bodies, the air flow around the object is not able to completely enclose the base of the object, and a low-pressure wake results which adds to the total drag on the object. Hoerner (1965) and Blevins (1992) are excellent references for information on base drag and aerodynamics in general.

The objective of this project was to collect experimental data of a shuttlecock and analyze different aerodynamic aspects of its flight. Surprisingly, there have been rather few other studies of the aerodynamic properties of a badminton shuttlecock. The shuttlecocks in current experiments were a low-cost type purchased from a local sporting goods stores, typical of what would be used in backyard games, and likely different from the professional quality shuttlecocks examined by Cooke. To perform theses experiment, the shuttlecocks were dropped in a free fall from the top floor inside of two different buildings on Bradley University's campus, and testing was also performed inside Bradley's subsonic wind tunnel. Two separate shuttlecocks of the same type were tested, and their masses, velocities, and areas were averaged for analysis. These results were then compared to those in the published literature on this topic. Details of the experimental methodology are given in the next section.

II. EXPERIMENTAL EQUIPMENT AND MEASUREMENT TECHNIQUE

The shuttlecocks tested were of the same modern construction, and utilized a cork hemispherical ball at the end of the plastic skirt. The diameters of the skirts on the shuttlecocks being tested were between 6.35 cm and 6.39 cm. These diameters were considered the characteristic diameter used in the analysis for scaling the Reynolds number. The masses of the shuttlecocks varied between 5.79 g and 5.95 g. Initially, the shuttlecocks were weighed on scales with an accuracy of +/- 0.005 g. The nose of the shuttlecock had a diameter of 26 mm. The shuttlecocks used in the present study were similar to the Tournament style analyzed by Cooke.

The density of indoor air in Peoria, IL can be found by using the ideal gas law, using room temperature of 20 °C. Peoria is about 600 ft (183 m) above sea level, and local atmospheric pressure is usually around 1.0 bar (0.99 atm).

$$\rho = \frac{PM}{RT} = \frac{(100000 Pa)(29 kg/kmol)}{(8314 J/kmol \cdot K)(293 K)} = 1.19 \frac{kg}{m^3} \tag{3}$$

A. Free-fall tests

The testing was conducted in two different locations: on the third floor of an atrium in a building with a large open area, and from the third floor of a stairwell in an adjacent building. Both locations were indoors and thus sheltered from winds. The stairwell used was chosen because it is four stories high. This extra distance allowed the shuttlecock to reach terminal velocity easily. The stairwell drop distance was measured and found to be 11.53 m. The terminal velocity of the two shuttlecocks was found using a radar gun with an accuracy of +/- 0.1 mph. The air movement in the atrium was considered negligible, and the dropping position of the shuttlecock was observed to be uniform. The freefall tests allowed the shuttlecock to begin rotating along its major axis as it began to fall. The data of the two shuttlecocks was averaged to allow for a general comparison between shuttlecocks. The value of the velocity approached a constant value both noticeably by visual observation and by the instantaneous value of the radar gun.

Thus terminal velocity of the shuttlecocks was assumed based on the final instantaneous velocity measured during the drop before impact on the floor. The validity of this assumption will be tested in the results and discussion section.

The shuttlecock was dropped from a height and clocked with the radar gun as it fell. It was insured that the shuttlecock was falling at its terminal velocity by observing that the instantaneous velocity was the same value as the peak velocity. In addition to the speed measurement, a time measurement for each fall was also taken with a stopwatch. Multiple trials were conducted and the data was averaged over all the experiments.

The Radar gun used was a STALKER ATS, with an accuracy of +/- 0.1 MPH (+/- 0.05 m/s), and a speed range of 1-300 MPH (1-480 KPH), which displays both the peak and the current velocity. It uses a 20 milliwatt Ka-band dual horn microwave signal. Its rated range for use with baseballs is 400-450 ft (122-137 m), well within the range needed for this testing.

B. Wind Tunnel Testing

Figure 1: Shuttlecock in Wind Tunnel

Figure 1 shows how the shuttlecock is mounted in the wind tunnel. Smoke from a simple fog machine is used to visualize the airflow past the shuttlecock. The wind tunnel is equipped with a load cell to measure the horizontal and vertical forces on the test object. The speed of the wind tunnel is variable and measured with a pitot tube. The wind tunnel was utilized to measure the drag force on the shuttlecock at various air speeds ranging from 24 mph to 110 mph. The shuttlecock was attached to a shaft and mounted in the wind tunnel, with the nose pointing directly into the wind. Before mounting the shuttlecock, the load cell was calibrated. Figure 2 shows the calibration of the load cell. The relationship between the actual drag force and the measured force given in Figure 2 was used to correct the data after it was collected.

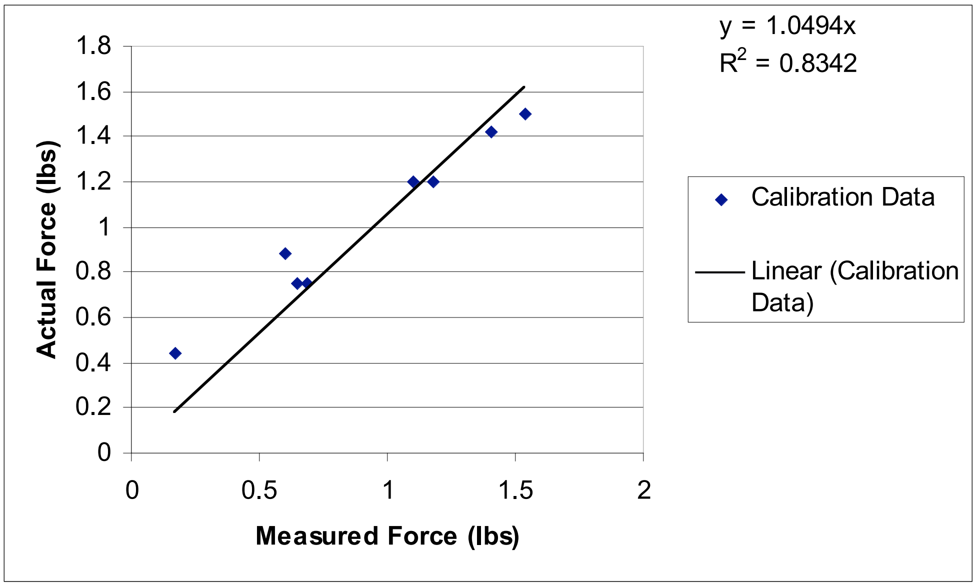

Figure 2: Calibration data for load cell used in wind tunnel testing.

The data from this speed range is pertinent as the velocity of the shuttlecock in a regular game. Therefore, the drag forces and coefficients obtained are the actual drag forces and coefficients the shuttlecock experiences when in use. The drag force measurement had an uncertainty of approximately +/- 0.2 N.

The Bradley University Subsonic Wind Tunnel has a test section of size 11" by 14" (28 cm by 36 cm), resulting in a cross area of 154 in^2, or 1008 cm^2. The blockage ratio is the ratio of the area of the test object to the area of the test section, equal to:

$$BR = \frac{A_{shuttlecock}}{A_{tunnel}} = \frac{\frac{\pi}{4}(6.37cm)^2}{1008cm^2} = 0.032 = 3.2\%$$

Blockage effects for a blockage ratio of less than 5% can generally be neglected (Barlow, 1999). The Bradley Subsonic Wind Tunnel is a standard open circuit wind tunnel design with flow straighteners and turbulence screens, and contraction ratio of 9:1 from the inlet to the test section.

III. RESULTS

Results from the freefall tests and the wind tunnel testing are presented separately.

A. Freefall Tests

The coefficient of drag was found by solving a force balance on the shuttlecock, and assuming the velocity recorded was indeed the terminal velocity. Terminal velocity is achieved in a freefall when an object has accelerated to the point where the drag force equals the weight of the object.

$$\sum F = F_D - W = ma = 0$$

Using the definition of drag coefficient from Equation (1), Newton's second law can be written as:

$$\frac{1}{2}C_D\rho AV_t^2 = mg$$

The area in this equation is the cross-sectional area of the shuttlecock skirt. The coefficient of drag (C_D) can be found in Equation 4.

$$C_D = \frac{2mg}{\rho AV^2} \quad (4)$$

In this equation, m is the mass of the shuttlecock, g is the gravitational acceleration, ρ is the density, and V is the terminal velocity. Subsequently, the Reynolds number at this velocity can be calculated using Equation 2.

The Drag Coefficient, C_D, and Reynolds Number were calculated for each terminal velocity obtained. These values were then averaged for the twenty trials completed. The averaged terminal velocity for the two different shuttlecocks was 6.79 m/s (15.2 mph), with a standard deviation of 0.07 m/s from the twenty trials. From the average velocity from the trials, the drag coefficient can be obtained from Equation 4, assuming the measured velocity is indeed the terminal velocity.

$$C_D = \frac{2\times(0.00585kg)\left(9.81\frac{m}{s^2}\right)}{\left(1.2\frac{kg}{m^3}\right)\left(\frac{\pi}{4}(0.0637m)^2\right)\left(6.79\frac{m}{s}\right)^2} = 0.65$$

$C_D = 0.65$ and the averaged Reynolds numbers for the two shuttlecocks was:

$$\text{Re} = \frac{\left(1.2\frac{kg}{m^3}\right)\times\left(6.79\frac{m}{s}\right)\times(0.0637m)}{\left(1.8*10^{-5}\frac{kg}{m\bullet s}\right)} = 28{,}900$$

B. Wind Tunnel Testing

Figure 3 shows the drag force with respect to air speed for the shuttlecock and the theoretical drag force for a sphere the size of the nose is also plotted for comparison. As expected from theory, the measured drag force is approximately proportional to the square of the air speed.

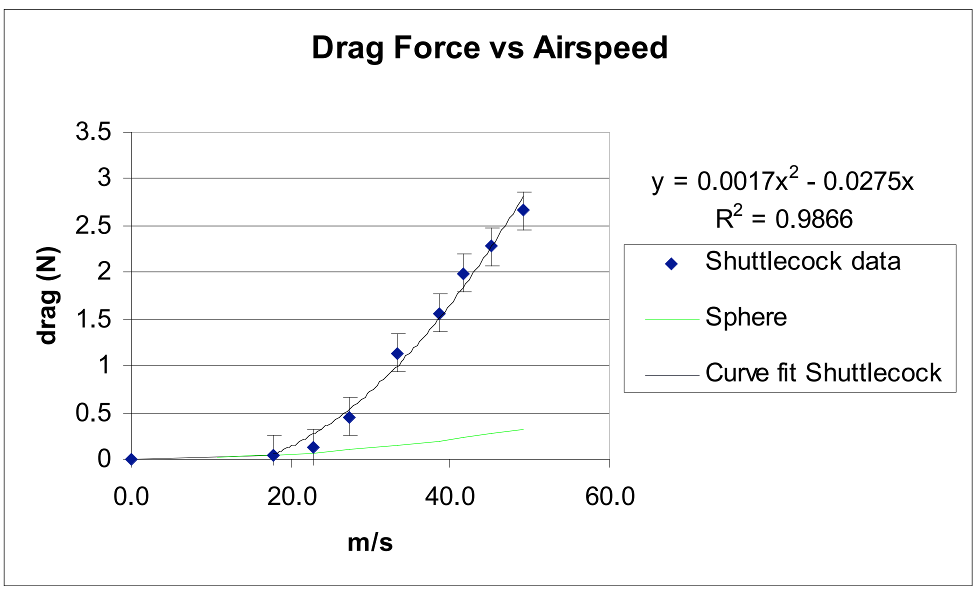

Figure 3: Drag force as a function of air speed for wind tunnel tests of shuttlecock.

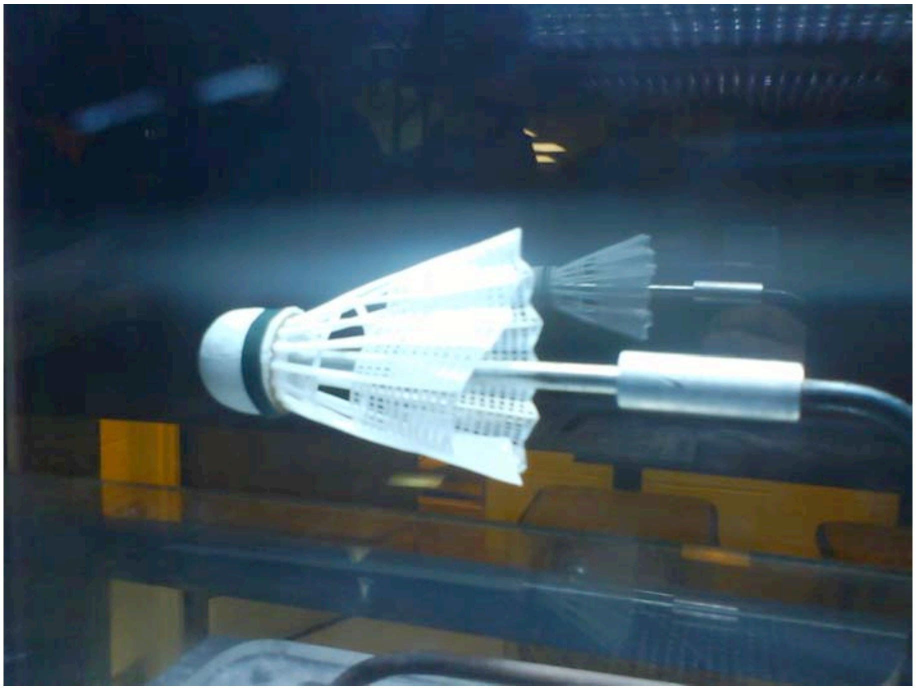

Figure 4: Shuttlecock Skirt Deformation.

Figure 4 shows a picture of the shuttlecock from the wind tunnel testing, in which deformation of the skirt can be clearly seen. While the shuttlecock is the fastest moving projectile in any game reaching speeds of up to approximately 200 mph, it also has some of the highest drag forces compared to any other projectile at those speeds. Figure 5 shows the measured values of drag coefficient as a function of the Reynolds number for the shuttlecock for the wind tunnel testing.

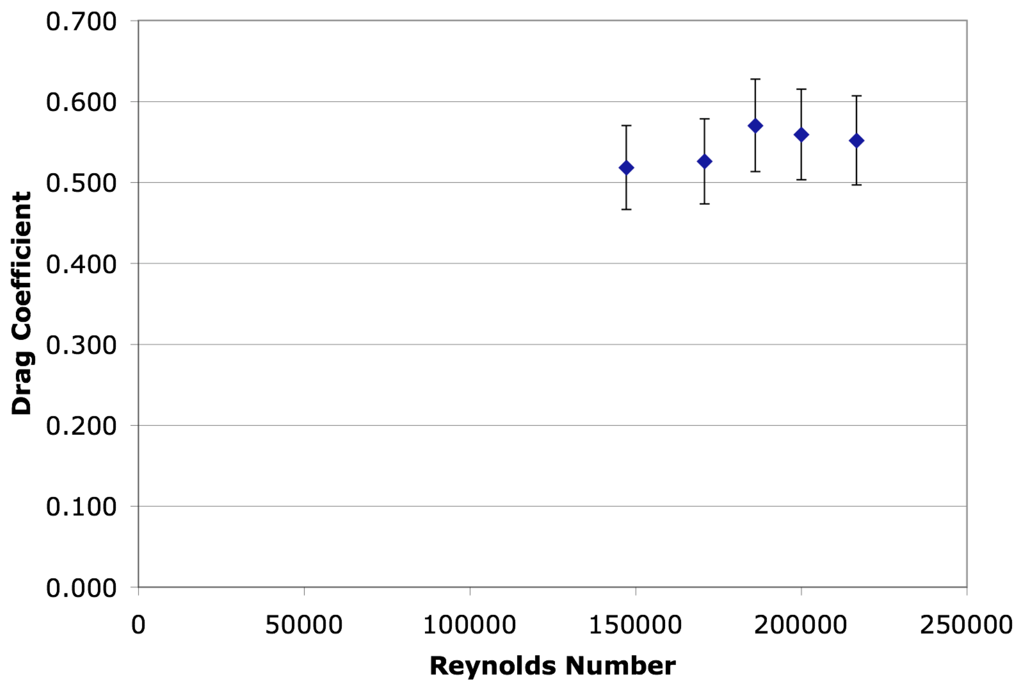

Figure 5: Drag Coefficient as a function of Reynolds Number

The drag coefficient for a shuttlecock is nearly constant over the range of Reynolds numbers tested, with an average value for the five data points of C_D = 0.55. It is interesting to note that this value is between that of a sphere and a cone all with the same diameter at these Reynolds numbers. The drag coefficient for the sphere is about 0.45 and for the cone at an angle of approximately 37° is 0.6 (Blevins). The error bars on the graph are based on the quantifiable uncertainty of the drag coefficient, as discussed in the next section.

C. Modeling

The transient 1-Dimensional trajectory of the shuttlecock in a freefall can be modeled using a force balance and Newton's second law.

$$\sum F = W - F_D = ma = m\frac{dV}{dt}$$

This equation can be re-arranged to solve for the rate of change of velocity:

$$mg - \frac{1}{2}C_D\rho AV^2 = m\frac{dV}{dt}$$

$$\frac{dV}{dt} = g - \frac{C_D\rho AV^2}{2m} \qquad (5)$$

This first-order non-linear ordinary differential equation can be solved with a numerical technique such as Euler's method or a Runge-Kutta method. The initial condition is that V(t=0) = 0. Inserting the numerical values into Equation 5 yields:

$$\frac{dV}{dt} = 9.81\frac{m}{s^2} - \left(0.212\frac{1}{m}\right)V^2$$

The plot of velocity as a function of time generated by solving this equation is shown in Figure 6. It can be seen that the velocity asymptotically approaches the terminal velocity of 6.79 m/s. In order to define when the terminal velocity has been reached, a criteria that the velocity was 99% of the terminal velocity (or 6.73 m/s). This occurs after 1.8 s, when the shuttlecock has fallen a distance of 9.1 m. This is less than the 11.5 m distance of the free fall tests, justifying the assumption that the final velocity recorded was in fact the terminal velocity, within 1% accuracy.

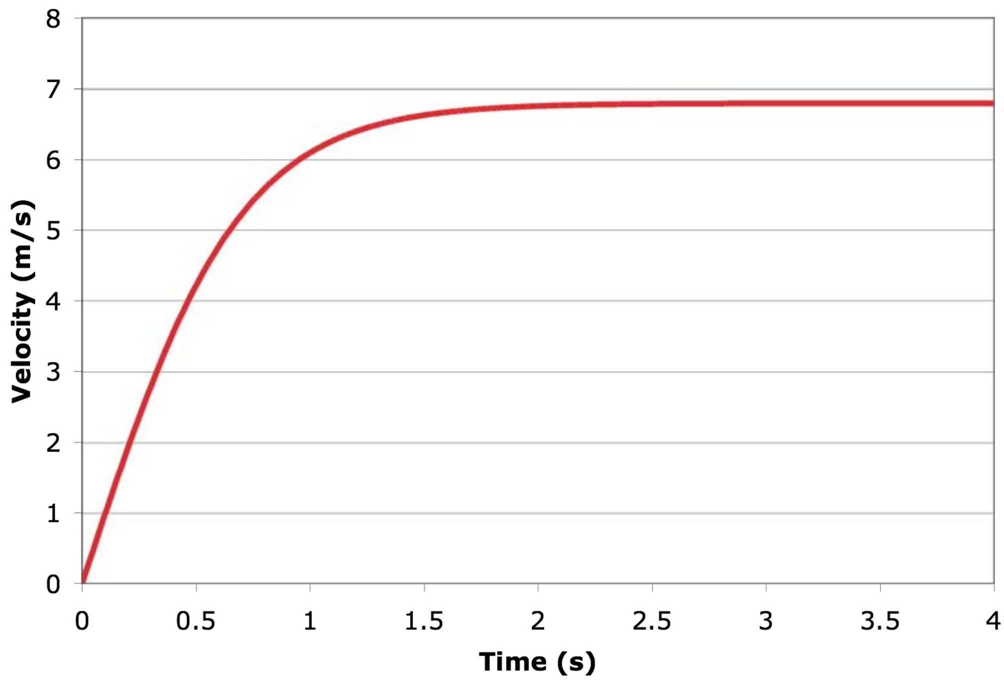

Figure 6. Calculated velocity as a function of time for a 1D simulation of the shuttlecock freefall experiment.

D. Error Analysis

The following error analysis was considered for the calculations of the coefficient of drag and Reynolds number. The size of the error bars shown in Figure 5 was calculated from an uncertainty analysis. The uncertainty in the calculated values was calculated based on the uncertainties in the measured quantities and a root-mean-squared propagation of errors model, such as Equation 6.

$$\Delta f = \sqrt{\sum_{\mathrm{i}} \left(\frac{\partial f}{\partial x_i} \Delta x_i \right)^2} \qquad (6)$$

Here *f* represents a calculated quantity (such as drag coefficient), and each x_i is a measured quantity, such as mass or diameter. Δf is the uncertainty in the calculated value, and $\Delta f/f$ would be the relative uncertainty in the calculated value, usually expressed as a percentage. For the free-fall tests, the primary source of experimental uncertainty was the uncertainty in the measured velocity. For the wind tunnel testing, the main contributor to the uncertainty of the drag coefficient was the uncertainty in the drag force. The other factors taken into consideration were the uncertainty in the diameter of the shuttlecock (+/- 2 mm), the uncertainty of the mass of the shuttlecock (+/- 0.05 g) and the uncertainty in the air speed of the wind tunnel (+/- 0.5 mph). The uncertainty in the air density was assumed to be negligible.

For the free fall tests the uncertainty of the velocity measured by the radar gun was taken to be +/- 0.07 m/s (+/- 0.2 mph) based on the variability of the recorded measured terminal velocities from the twenty trials. This is larger than the manufacturer's specified accuracy of +/- 0.1 mph.

$$\frac{\Delta C_D}{C_D} = \sqrt{\left(\frac{\Delta M}{M} \right)^2 + 4\left(\frac{\Delta V}{V} \right)^2 + 4\left(\frac{\Delta D}{D} \right)^2} \qquad (7)$$

$$\frac{\Delta C_D}{C_D} = \sqrt{\left(\frac{0.05}{5.85}\right)^2 + 4\left(\frac{0.07}{6.79}\right)^2 + 4\left(\frac{0.05}{6.37}\right)^2} = 2.77\%$$

$$\frac{\Delta R_e}{R_e} = \sqrt{\left(\frac{\Delta V}{V}\right)^2 + \left(\frac{\Delta D}{D}\right)^2} \qquad (8)$$

$$\frac{\Delta R_e}{R_e} = \sqrt{\left(\frac{0.07}{6.79}\right)^2 + \left(\frac{0.05}{6.37}\right)^2} = 1.32\%$$

For the wind tunnel tests the formula for the uncertainty in the measured drag coefficient is:

$$\frac{\Delta C_D}{C_D} = \sqrt{\left(\frac{\Delta F}{F}\right)^2 + 4\left(\frac{\Delta V}{V}\right)^2 + 4\left(\frac{\Delta D}{D}\right)^2}$$

The values of the measured drag force fluctuate quite a bit, and the uncertainty in the value read from the digital display is +/- 0.2 N.

$$\frac{\Delta C_D}{C_D} = \sqrt{\left(\frac{0.2}{2.0}\right)^2 + 4\left(\frac{0.5}{90}\right)^2 + 4\left(\frac{0.05}{6.37}\right)^2} = 10.2\%$$

Thus in summary, the drag coefficient measured in the wind tunnel tests can be stated to be C_D = 0.55 +/- 0.05, and the drag coefficient from the free-fall tests is C_D = 0.65 +/- 0.02.

IV. DISCUSSION

The drag properties for badminton shuttlecocks have not been widely tested. *Shuttlecock Aerodynamics* by Alison J. Cooke compared the drag coefficients for feather shuttlecocks to synthetic shuttlecocks. She found that the coefficient of drag averaged to be around 0.48 for Reynolds numbers between 13,000 and 190,000 for a championship tournament shuttlecock. Wind tunnel and freefall testing were performed in the current investigation to measure the drag coefficient of shuttlecocks. Cooke's average value of C_D = 0.48 is lower than the values of 0.55 to 0.65 found in this study. Some contributing factors to this variation could include the difference in material or design of the shuttlecock, method of testing, and the accuracy of the instruments used during testing, as our shuttlecock is one that can be found at a local sporting goods store, and is most likely of a lesser quality than the one used in Cooke's study. It should also be note that in the free-fall tests the shuttlecocks can spin, while in the wind tunnel they were rigidly mounted and could not spin. This spinning motion could cause additional drag and explain why the drag coefficients from the freefall tests were higher than those form the wind tunnel testing. Since both the wind tunnel and freefall tests had higher drag coefficients than those measured by Cooke, it seems likely that the cheap synthetic shuttlecocks tested here, which are representative of what the typical recreational user would use, have a higher drag coefficient than the professional shuttlecocks analyzed by Cooke.

Worldwide, wind tunnel testing is the standard method to measure the coefficient of drag of an object. In this study, in addition to wind tunnel testing, free-fall tests with a high-accuracy radar gun were also used. Another possible method to ascertain drag coefficients is to use motion capture. With a digital video camera mounted on a tripod, a recording of the two-dimensional shuttlecock trajectory could be captured. Then image analysis software can be used to find the position of the shuttlecock in each image.

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