Sports Aerodynamics: Cricket Ball & Badminton Shuttlecock

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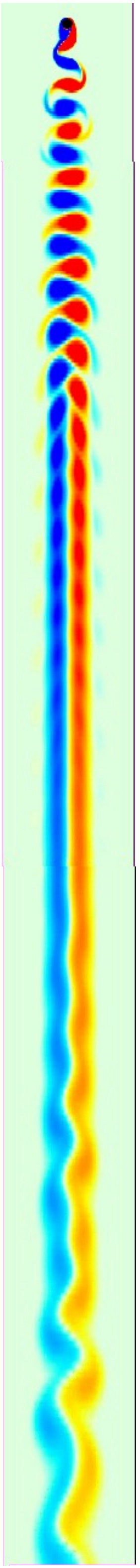
Acknowledgement: Students, Colleagues, Various funding agencies



Incompressible flow equations

Continuity Equation:

$$\boldsymbol{\nabla} \cdot \mathbf{u} = 0 \qquad on\ \Omega \ \times \ (0,T)$$

Momentum Equation:

$$\rho\left(\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \boldsymbol{\nabla}\mathbf{u} - \mathbf{f}\right) - \boldsymbol{\nabla} \cdot \boldsymbol{\sigma} = 0 \qquad on\ \Omega \ \times \ (0,T)$$

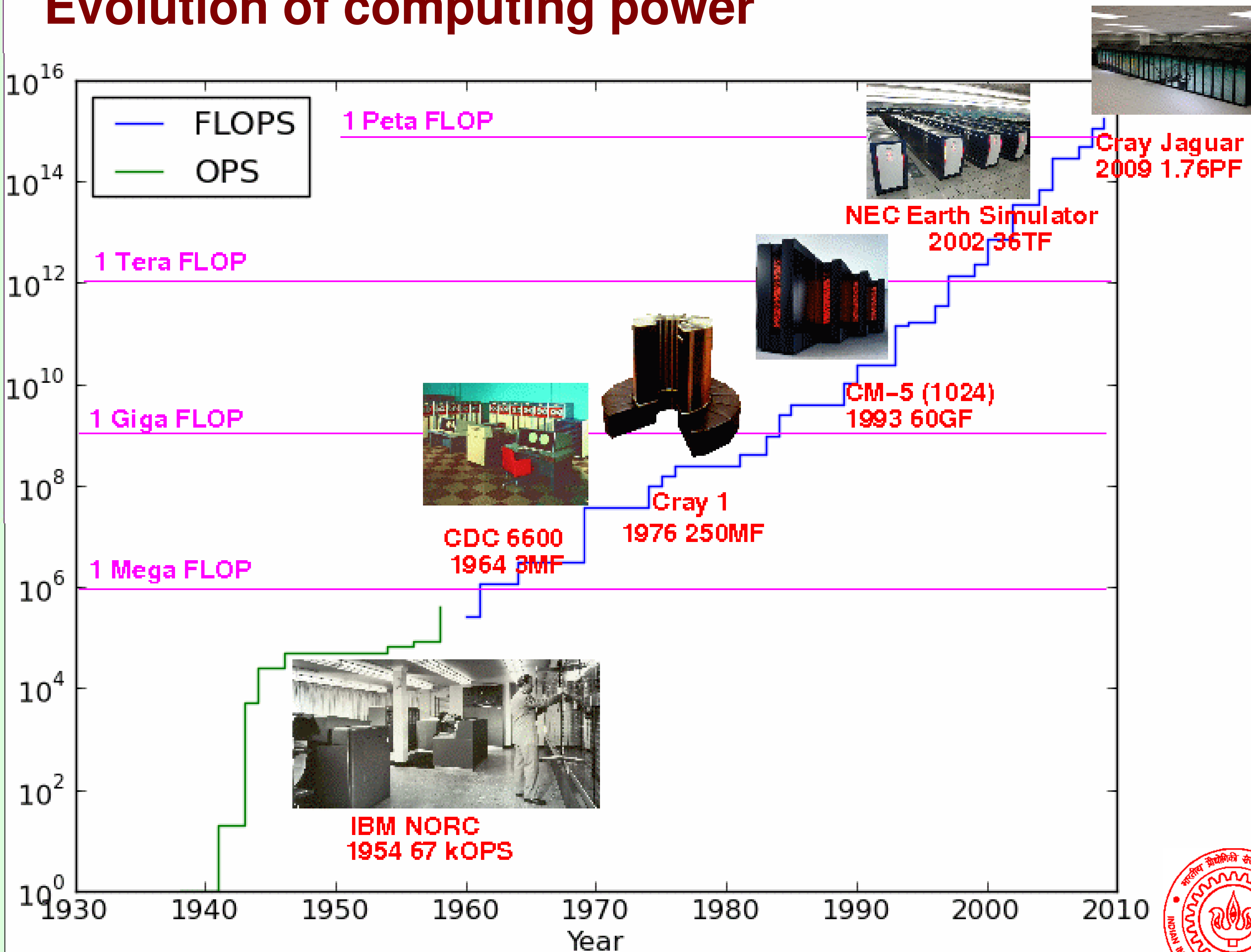
$$\boldsymbol{\sigma} = -p\mathbf{I} + \mathbf{T}\ ,\ \ \mathbf{T} = 2\mu\boldsymbol{\epsilon}(\mathbf{u})\ ,\ \ \boldsymbol{\epsilon}(\mathbf{u}) = \frac{1}{2}((\nabla\mathbf{u}) + (\nabla\mathbf{u})^T)$$

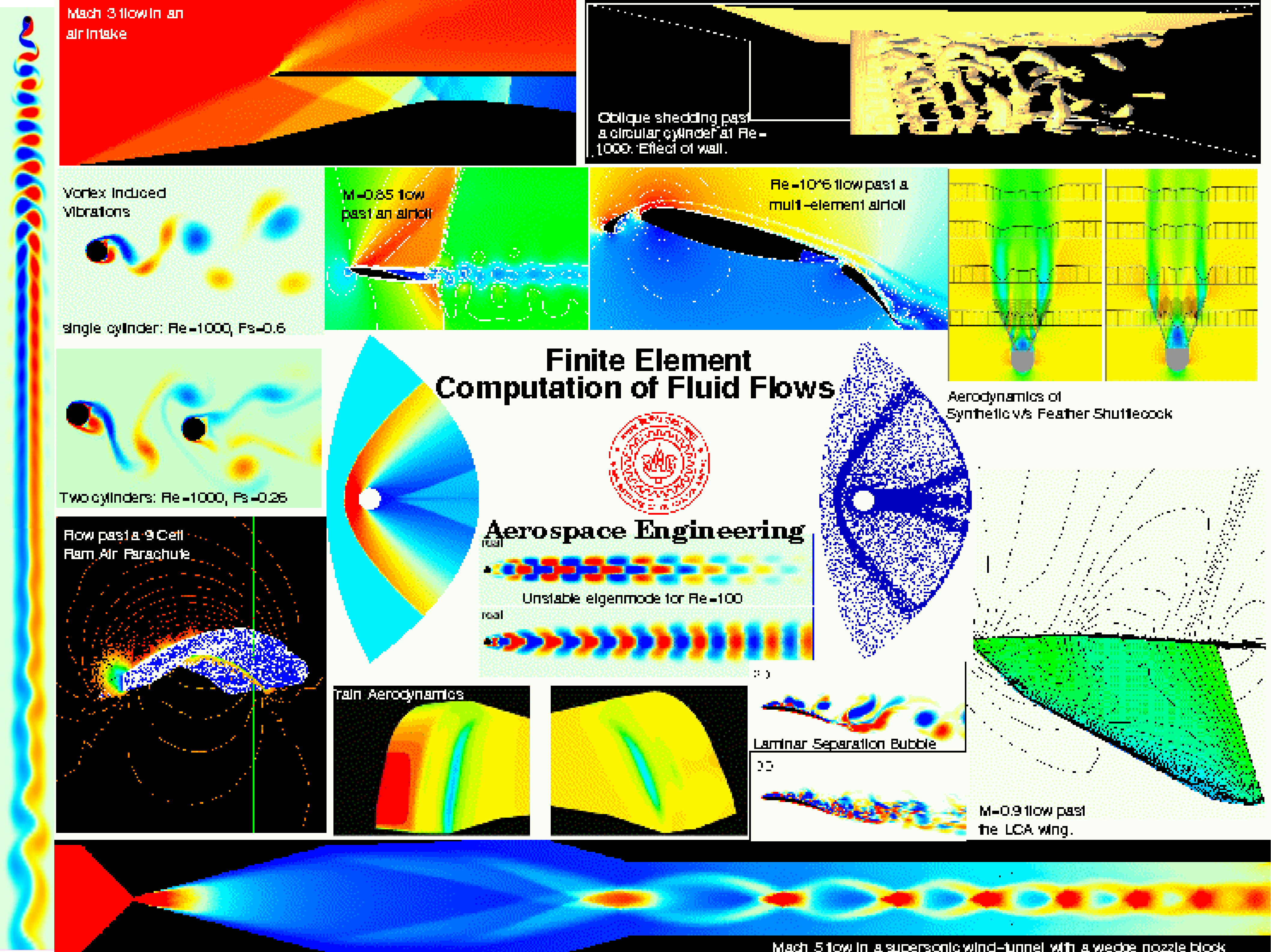
Boundary conditions & Initial condition:

$$\mathbf{u} = \mathbf{g}\ on\ \Gamma_g\ ,\ \ \mathbf{n} \cdot \boldsymbol{\sigma} = \mathbf{h}\ on\ \Gamma_h$$
$$\mathbf{u}(\mathbf{x}, 0) = \mathbf{u}_0\ on\ \Omega$$
$$\Gamma = \Gamma_g \cup \Gamma_h$$

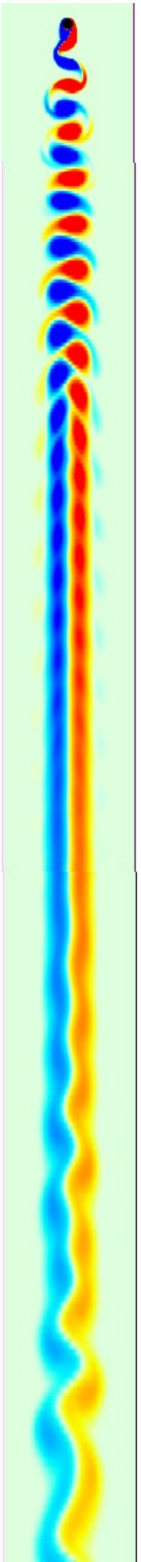
Unsteady, Non-linear, Coupled, PDE's

Evolution of computing power





Mach 3 flow in an air intake
Oblique shedding past a circular cylinder at Re = 1000: Effect of wall.
Vortex Induced Vibrations
single cylinder: Re=1000, Fs=0.6
M=0.85 flow past an airfoil
Re=10^6 flow past a multi-element airfoil
Finite Element Computation of Fluid Flows
Aerodynamics of Synthetic v/s Feather Shuttlecock
Two cylinders: Re=1000, Fs=0.26
Aerospace Engineering
real
Unstable eigenmode for Re=100
real
Flow past a 9 Cell Ram Air Parachute
Train Aerodynamics
Laminar Separation Bubble
M=0.9 flow past the LCA wing.
Mach 5 flow in a supersonic wind-tunnel with a wedge nozzle block



Finite Element Formulation (DSD/ST):

Given $(\mathbf{u}^h)_{n^-}$, find $\mathbf{u}^h \in (\mathcal{S}^h_{\mathbf{u}})_n$ and $p^h \in (\mathcal{S}^h_p)_n$ such that

$\forall \mathbf{w}^h \in (\mathcal{V}^h_{\mathbf{u}})_n,\ q^h \in (\mathcal{V}^h_p)_n,$

$$\int_{Q_n} \mathbf{w}^h \cdot \rho \left(\frac{\partial \mathbf{u}^h}{\partial t} + \mathbf{u}^h \cdot \boldsymbol{\nabla} \mathbf{u}^h - \mathbf{f} \right) d\Omega$$

$$+ \int_{Q_n} \boldsymbol{\varepsilon}(\mathbf{w}^h) : \boldsymbol{\sigma}(p^h, \mathbf{u}^h) dQ + \int_{Q_n} q^h \boldsymbol{\nabla} \cdot \mathbf{u}^h dQ$$

$$+ \sum_{e=1}^{n_{el}} \int_{Q_n^e} \frac{1}{\rho} \tau \left[\rho \left(\frac{\partial \mathbf{w}^h}{\partial t} + \mathbf{u}^h \cdot \boldsymbol{\nabla} \mathbf{w}^h \right) - \boldsymbol{\nabla} \cdot \boldsymbol{\sigma}(q^h, \mathbf{w}^h) \right] .$$

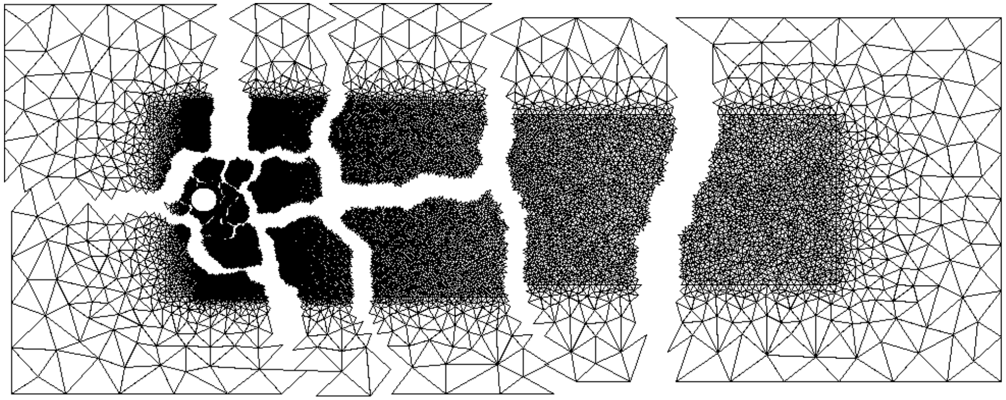
$$\left[\rho \left(\frac{\partial \mathbf{u}^h}{\partial t} + \mathbf{u}^h \cdot \boldsymbol{\nabla} \mathbf{u}^h - \mathbf{f} \right) - \boldsymbol{\nabla} \cdot \boldsymbol{\sigma}(p^h, \mathbf{u}^h) \right] dQ$$

$$+ \sum_{e=1}^{n_{el}} \int_{Q_n^e} \delta \boldsymbol{\nabla} \cdot \mathbf{w}^h \rho \boldsymbol{\nabla} \cdot \mathbf{u}^h dQ$$

$$+ \int_{\Omega_n} (\mathbf{w}^h)_n^+ . \rho \left((\mathbf{u}^h)_n^+ - (\mathbf{u}^h)_n^- \right) d\Omega = \int_{(P_n)_h} \mathbf{w}^h \cdot \mathbf{h}^h dP$$

Parallel Computing:

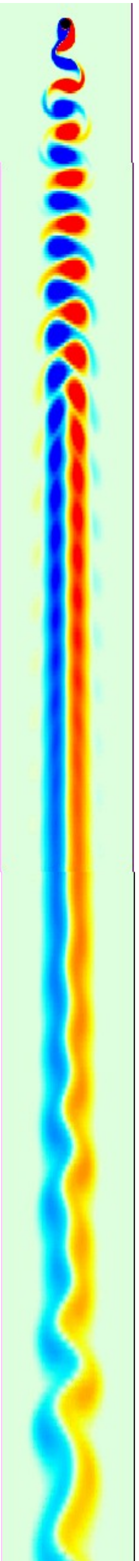
32 node Linux Cluster.
Each node:
Dual processor 3.06 Ghz Xeon,
512 K, 4 GB RAM, 72 GB HDD
Gigabit Switch



Domain partitioning

The non-linear equations resulting from the finite element discretization are solved using GMRES method with diagonal preconditioner

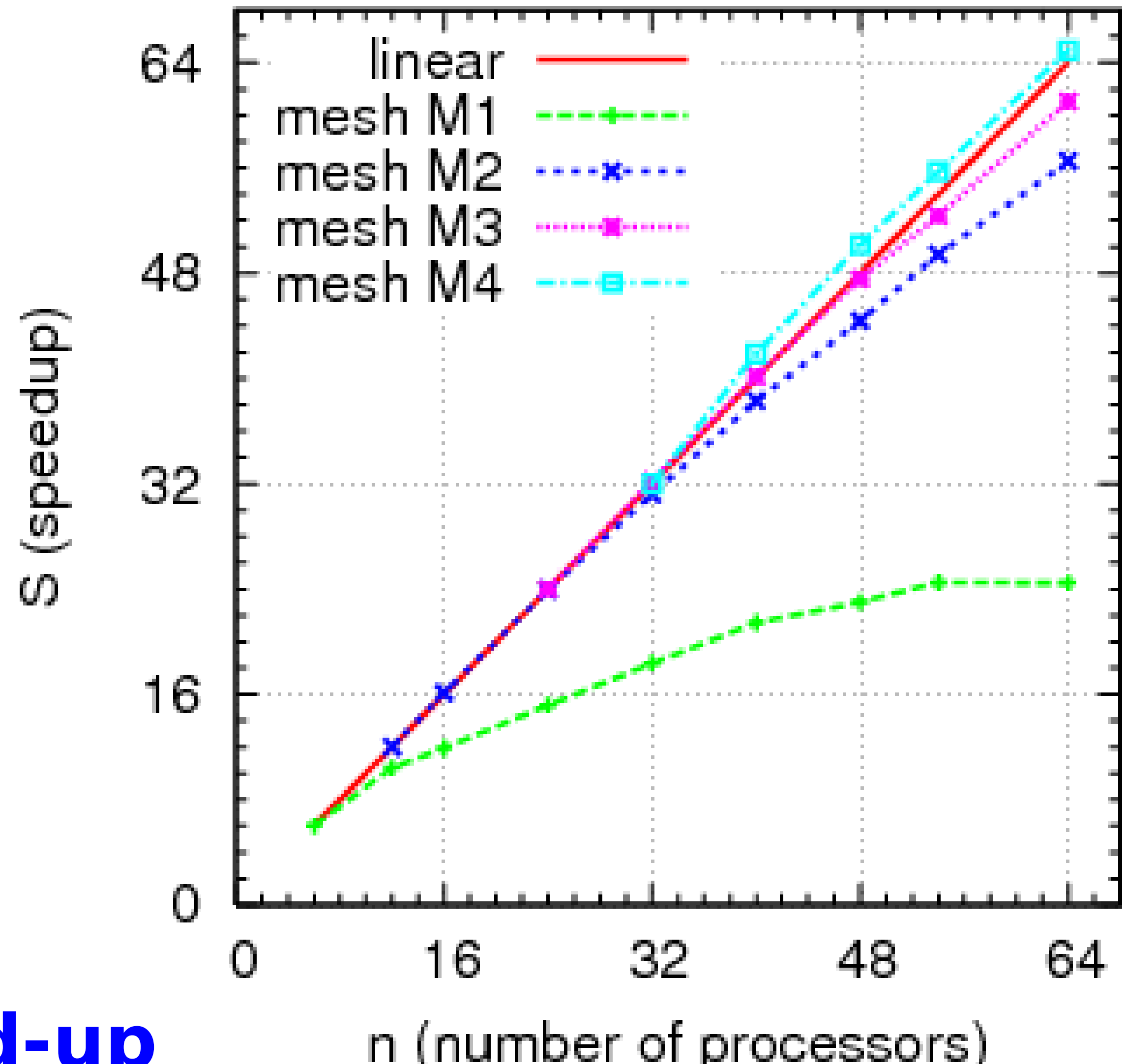
Using MPI Libraries



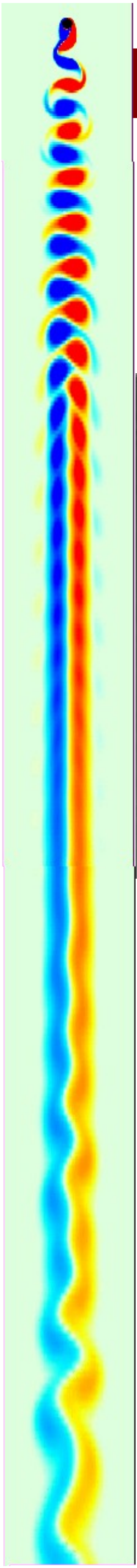
Parallel computing:

Mesh Statistics:
in millions

Mesh	nn	ne	neq
M1	1.3	2.3	4.9
M2	14.1	27.8	55.7
M3	28.0	55.6	111.4
M4	44.3	88.0	176.2

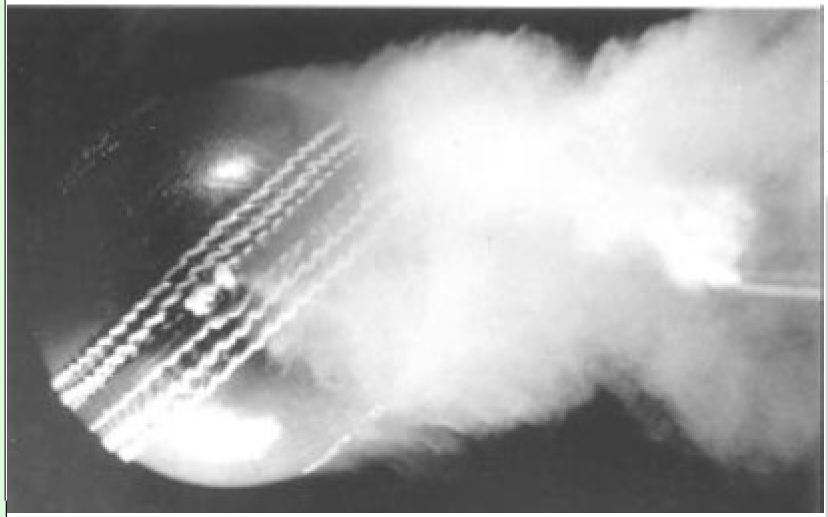


Super-linear Speed-up
Behara & Mittal, Parallel Computing (2009)

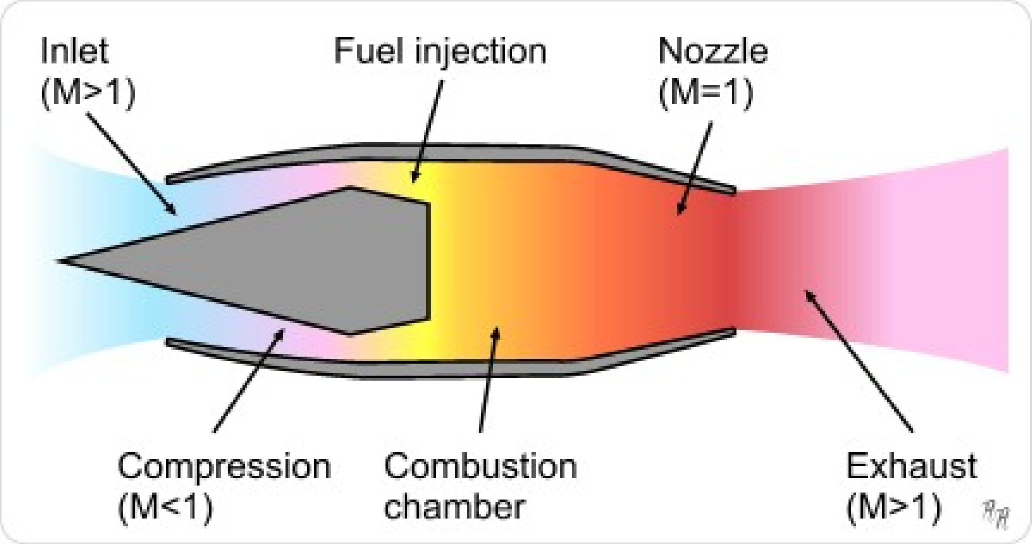


Menu for today's presentation

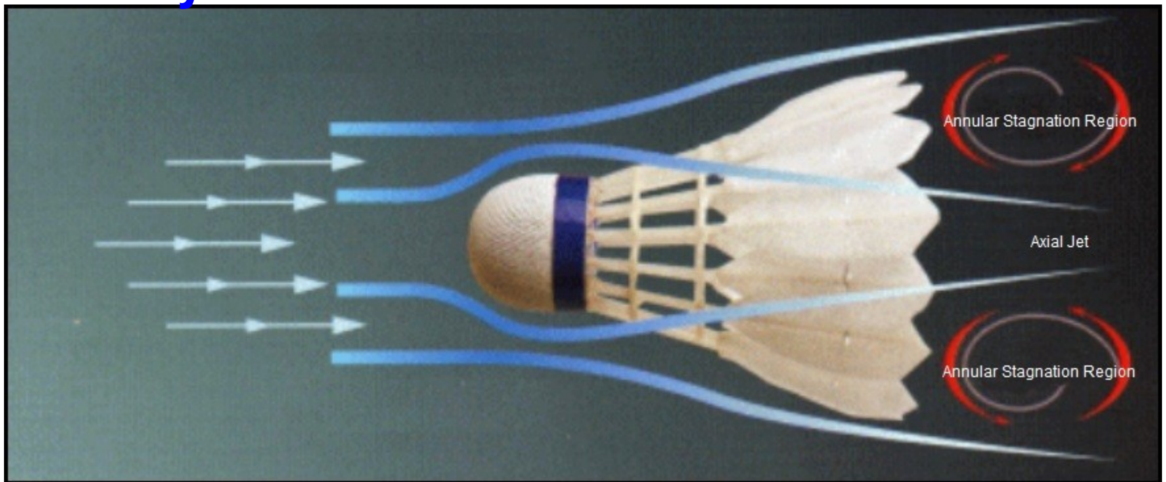
Aerodynamics of swing and reverse swing via ideas of Bluff Body Flows

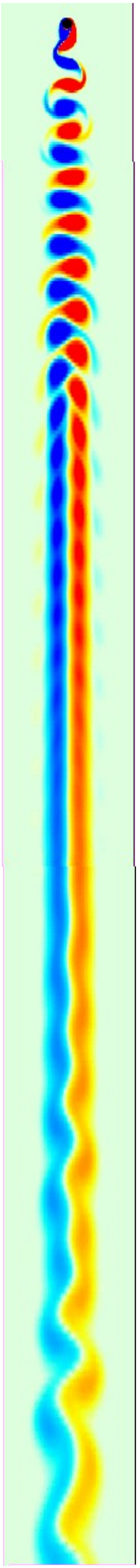


Aerodynamics of the Air intake of a Ramjet engine



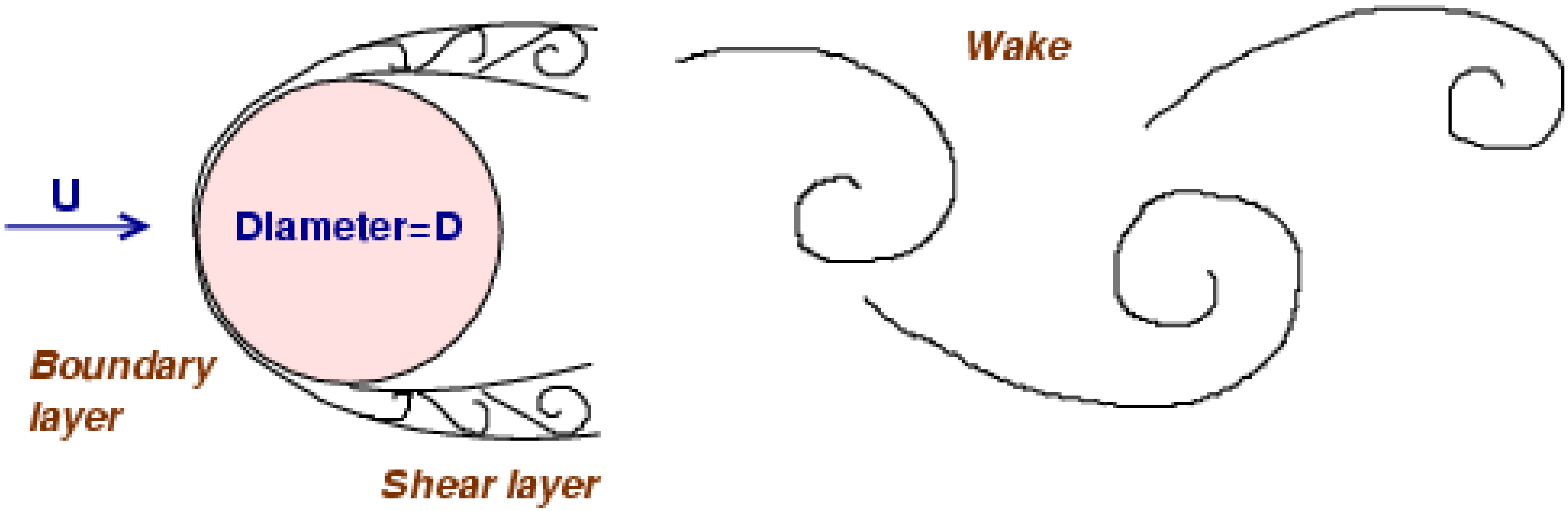
Aerodynamics of Birdie





Flow past a circular cylinder

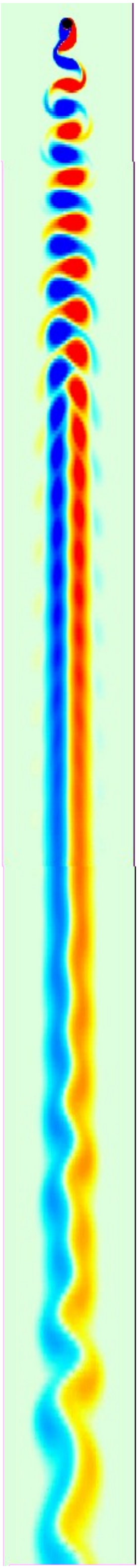
Cylinder: bluff body with simple geometry has all the flow complexities



Reynolds number=UD/ν

Williamson (1996)

Flow past a circular cylinder

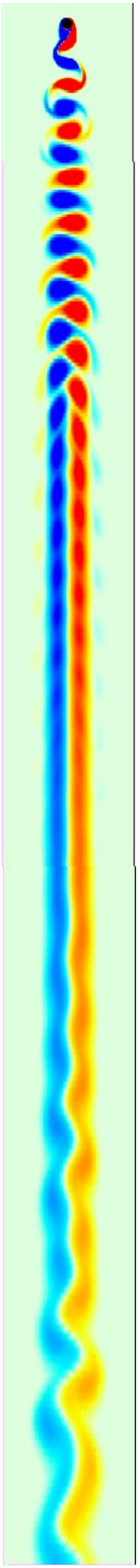


First convective wake instability: Re ~ 5; Monkewitz (1988)
Re ~ 4; (Mittal & Kumar, POF 2007)

Onset of flow separation: Re=6.29
Sen, Mittal, Biswas; JFM(2009)

First wake instability (self sustained) : Re ~ 47:
leads to von Karman shedding
Kumar, Mittal (IJNMF, CMAME (2006))

Transition of the wake (Mode A & B) :
Re ~ 150; Roshko (1954)
Re ~ 165; Norberg (1994)
Re ~ 160; Zhang, et al (1995)
Re ~ 188.5; Barkley & Henderson (1996)
Re ~ 205; Miller & Williamson (1994)
Re ~ 198; (Behara & Mittal PoF, 2010a,b)



Flow past a circular cylinder

A new mode of instability for the 2D wake:

Re ~ 110; (Verma & Mittal, PoF, 2011)

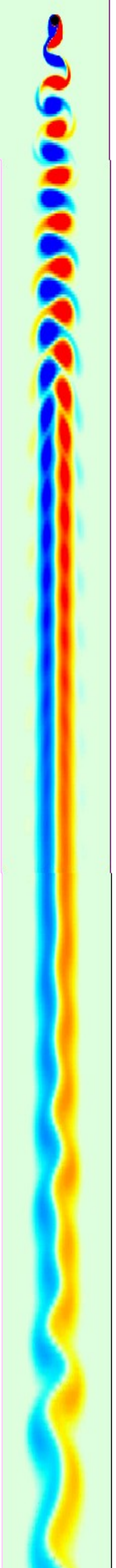
Shear layer instability (convective) : Wide scatter

Re ~ 1300; Bloor (1964)
Re ~ 350; Gerrard (1978)
Re ~ 1900; Unal & Rockwell (1988)
Re ~ 1200; Prasad & Williamson (1997)
Re ~ 740; Rajagopalan & Antonia (2005)
Re ~ 54; (Mittal et al., PoF, JFM, 2008)

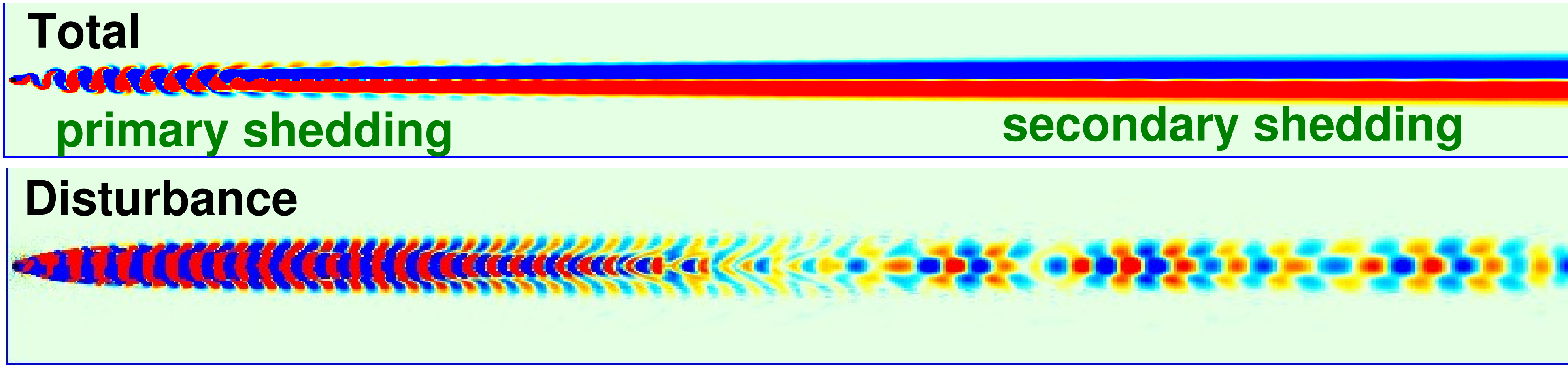
Secondary instability in the far wake (convective) :
Kumar & Mittal, JFM, 2012

We now investigate transition of the boundary layer

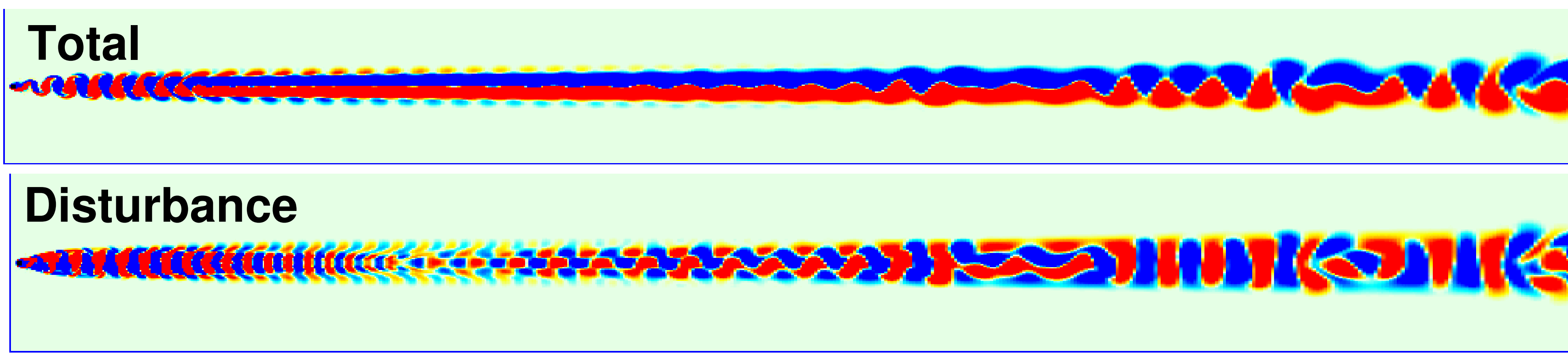
Total v/s Disturbance (unsteady – time-averaged) vorticity field:



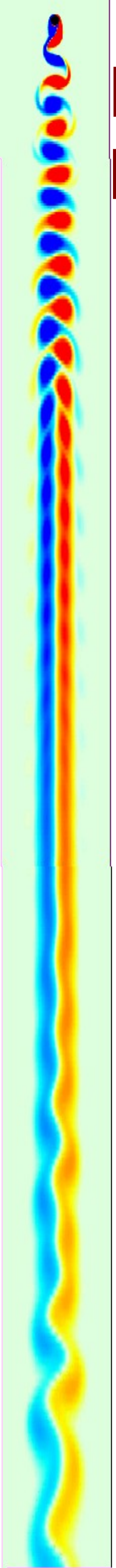
Re=150



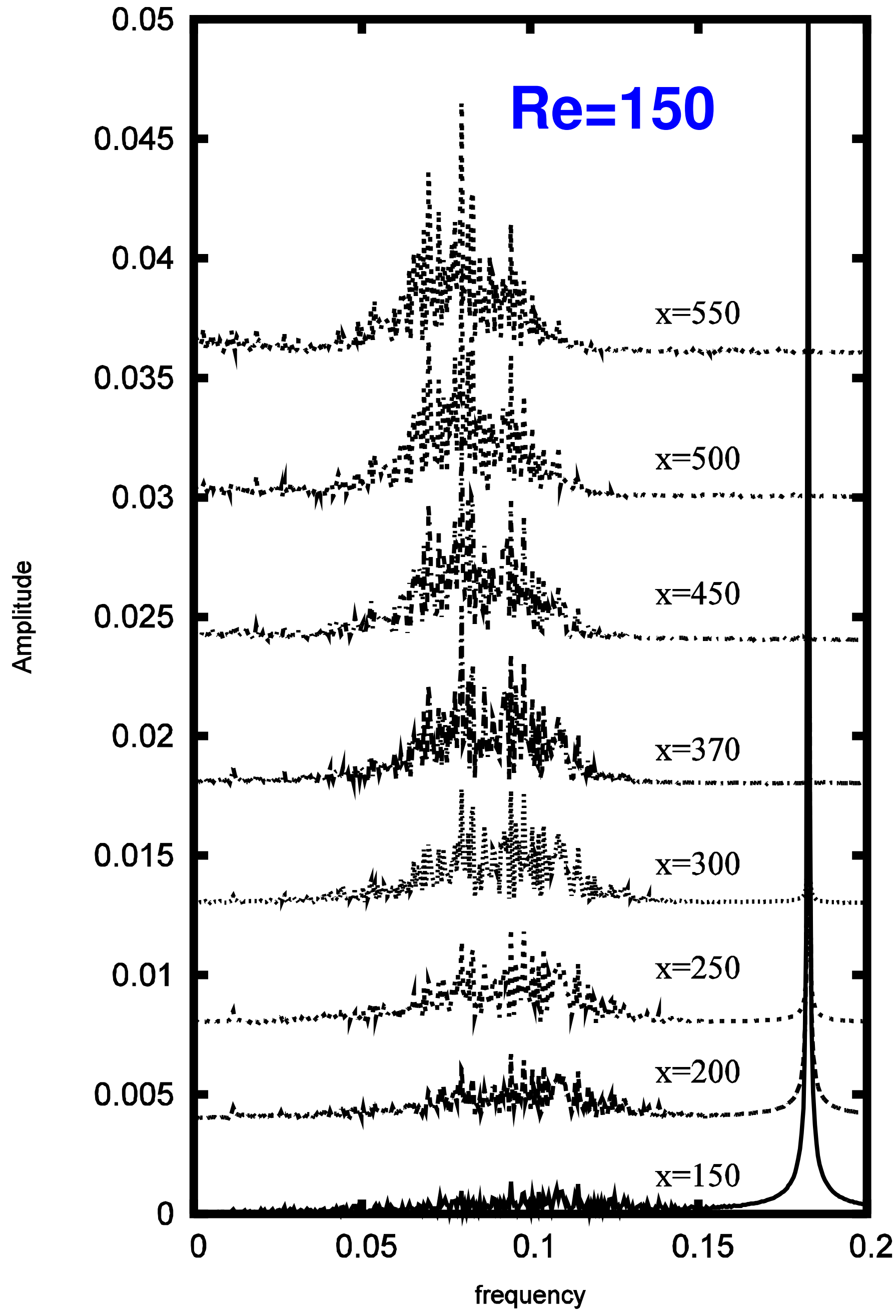
Re=200

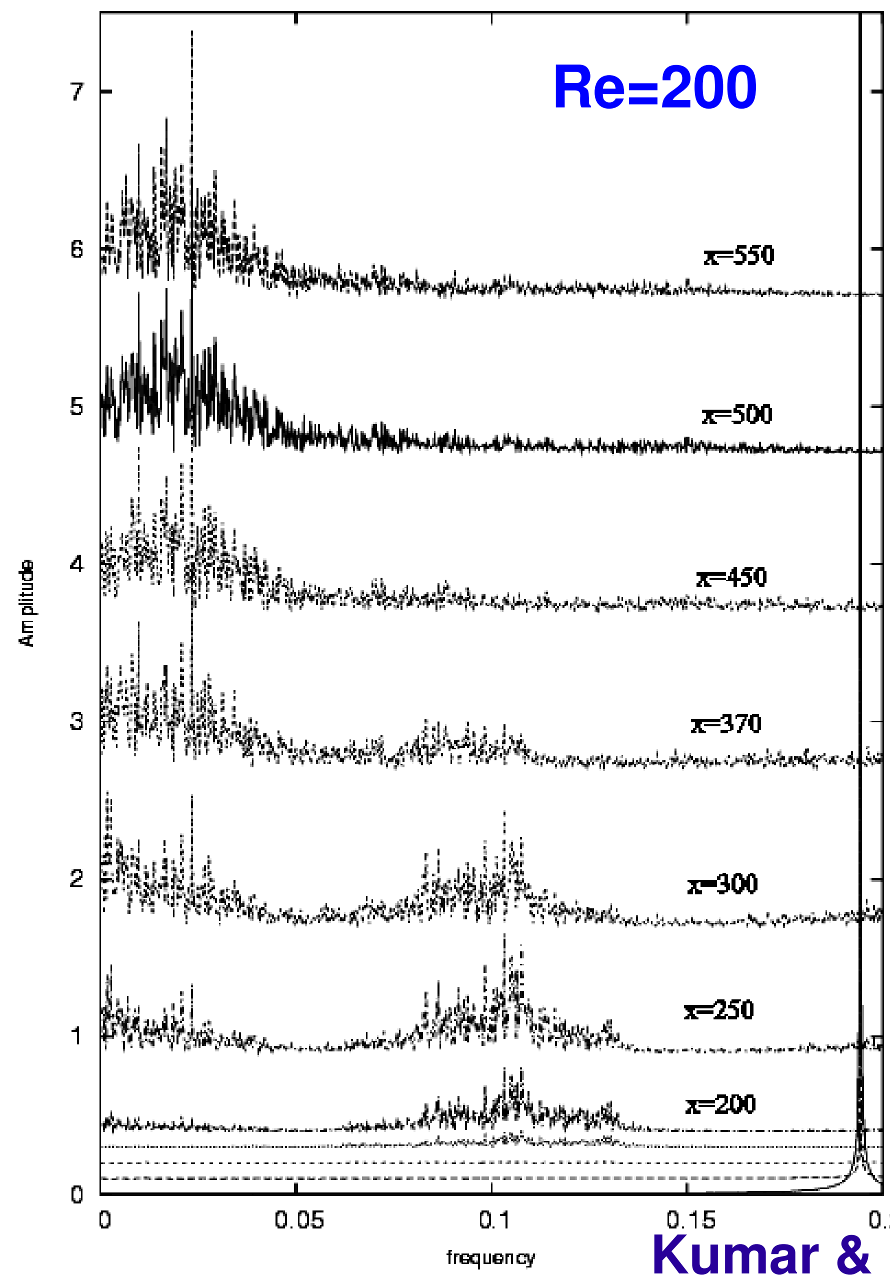


Kumar & Mittal, JFM (2012)

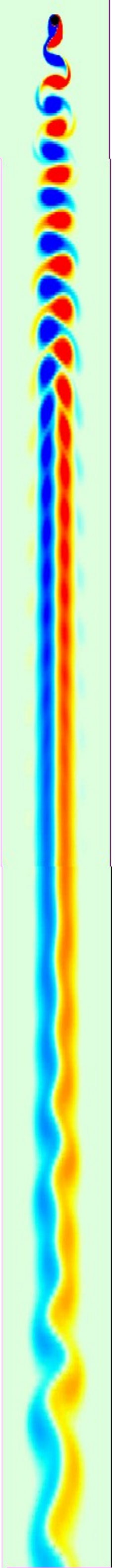


Frequency spectra of disturbance time history at various x locations





Kumar & Mittal, JFM (2012)



What causes secondary shedding ?

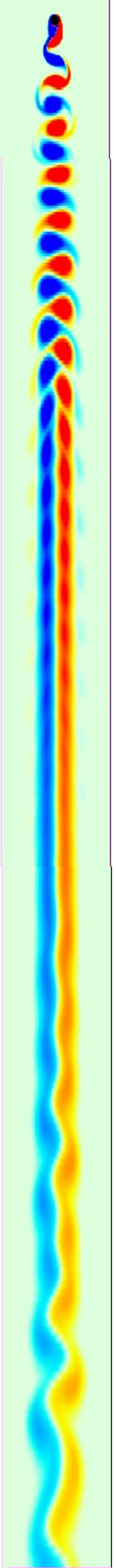
In the literature we find several conjectures:

Hydrodynamic instability (Taneda, J.Phy.Soc. Japan, 1959, Cimbala, Naguib and Roshko, JFM, 1988)

Non-linear interaction of free stream disturbances and the primary mode (Williamson and Prasad, JFM, 1993)

Vortex Pairing Mechanism(Matsui and Okude, Seventh Biennial Symposium on Turbulence, Rolla, Missouri, 1981)

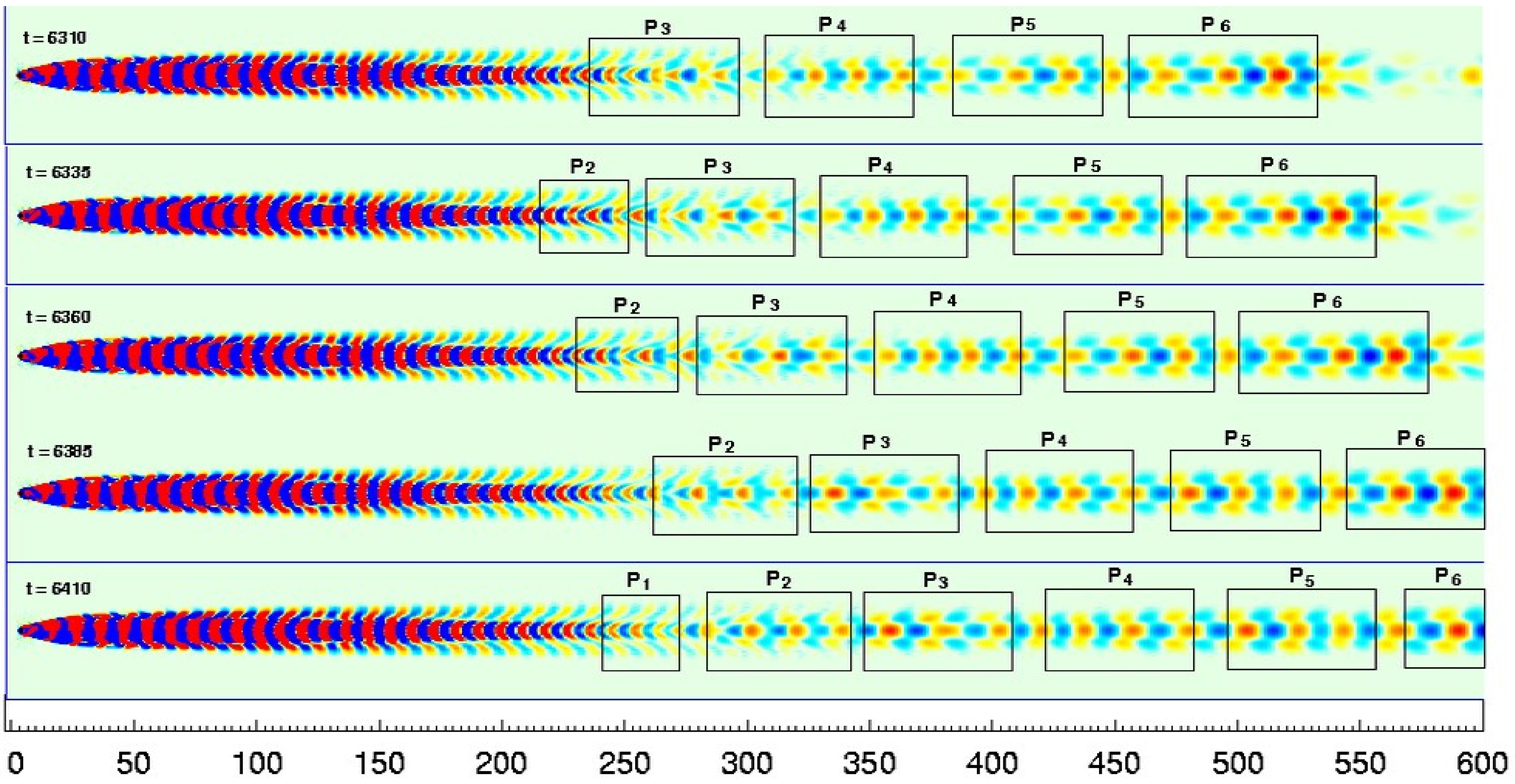
Kumar & Mittal, JFM (2012)



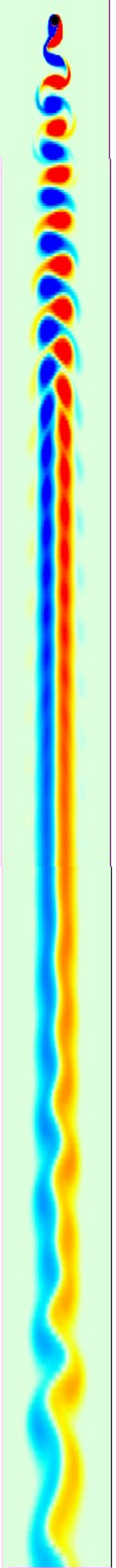
Disturbance (unsteady – time-averaged) vorticity field: Re=150

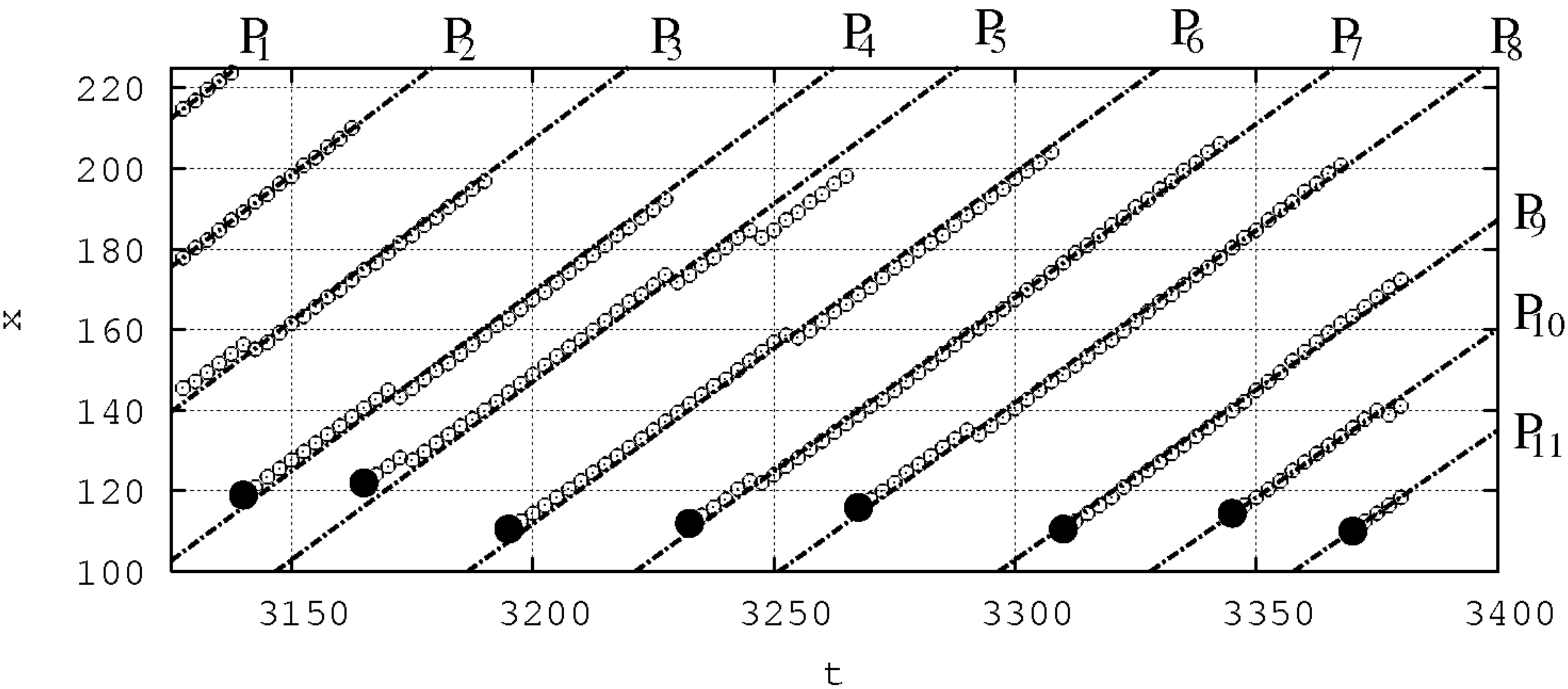
Observation: secondary shedding is comprised of disturbance packets propagating in the streamwise direction

primary disturbance **secondary disturbance**



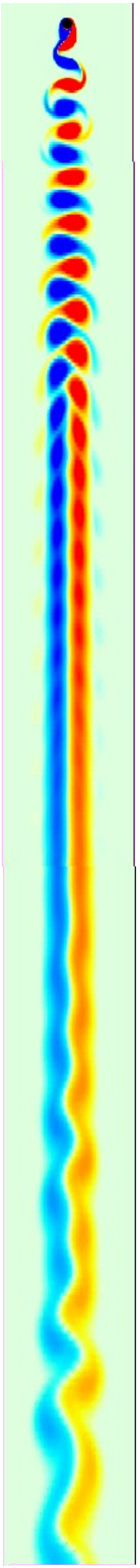
Disturbance (unsteady – time-averaged) vorticity field: Re=150





The packets travels at ~0.8-0.9 U

Kumar & Mittal, JFM (2012)



Linear Stability Analysis: convective instabilities

Consider two frames of reference:

- **x**: laboratory frame, fixed to the body
- **z**: frame moving with speed **c**

Transformation between the two frames:

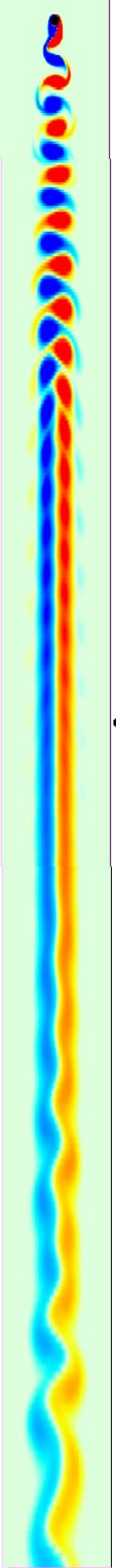
$$\mathbf{x} = \mathbf{z} + \mathbf{c}t, \quad \nabla_{\mathbf{x}} = \nabla_{\mathbf{z}}, \quad \left.\frac{\partial}{\partial t}\right|_{\mathbf{x}} = \left.\frac{\partial}{\partial t}\right|_{\mathbf{z}} - \mathbf{c}\cdot\nabla_{\mathbf{z}}$$

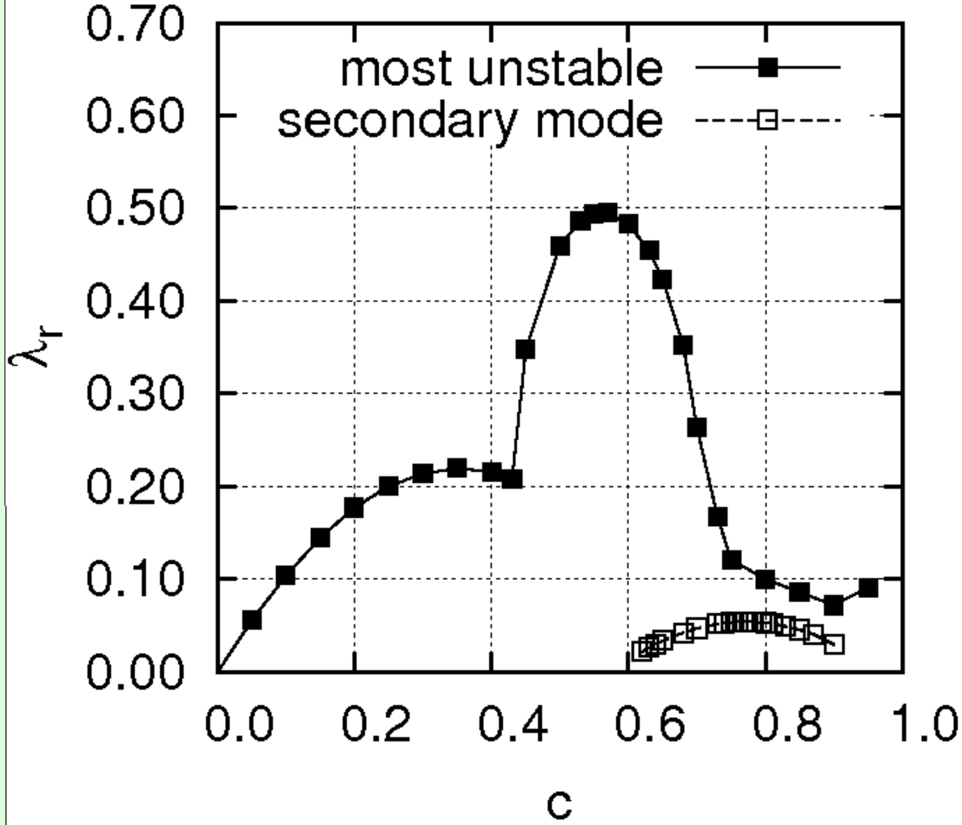
- Rewrite the equations in the moving frame.
  Choose a disturbance that moves with frame z (with speed c) and, therefore, appears to be stationary in that frame.

$$\mathbf{u}'(\mathbf{x},t) = \hat{\mathbf{u}}(\mathbf{x}-\mathbf{c}t)e^{\lambda t}, \qquad p'(\mathbf{x},t) = \hat{p}(\mathbf{x}-\mathbf{c}t)e^{\lambda t}.$$

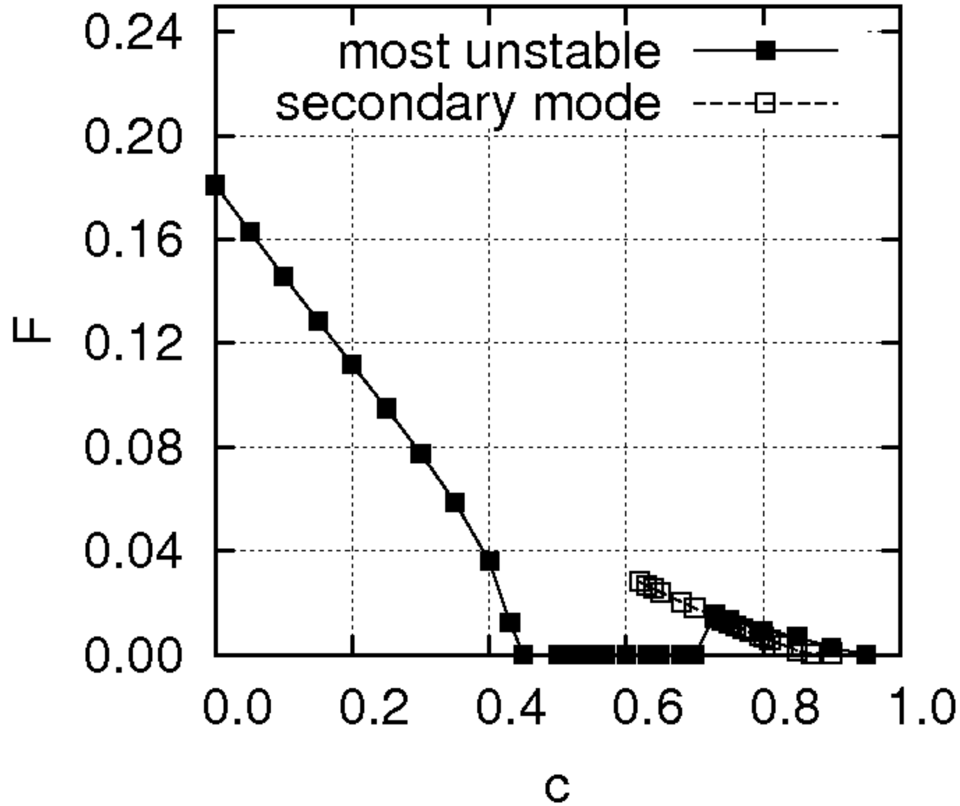
Mittal & Kumar POF (2007)

Linear Stability Analysis: Re=150, Time-averaged flows



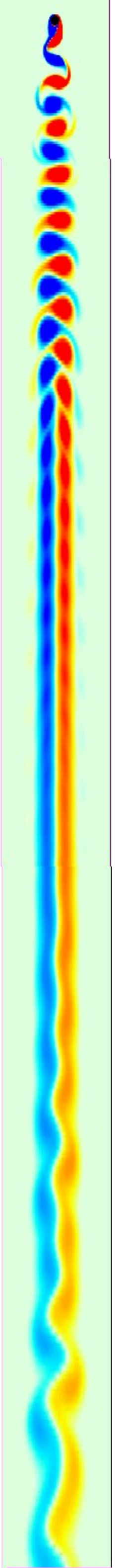


Growth rate



Frequency

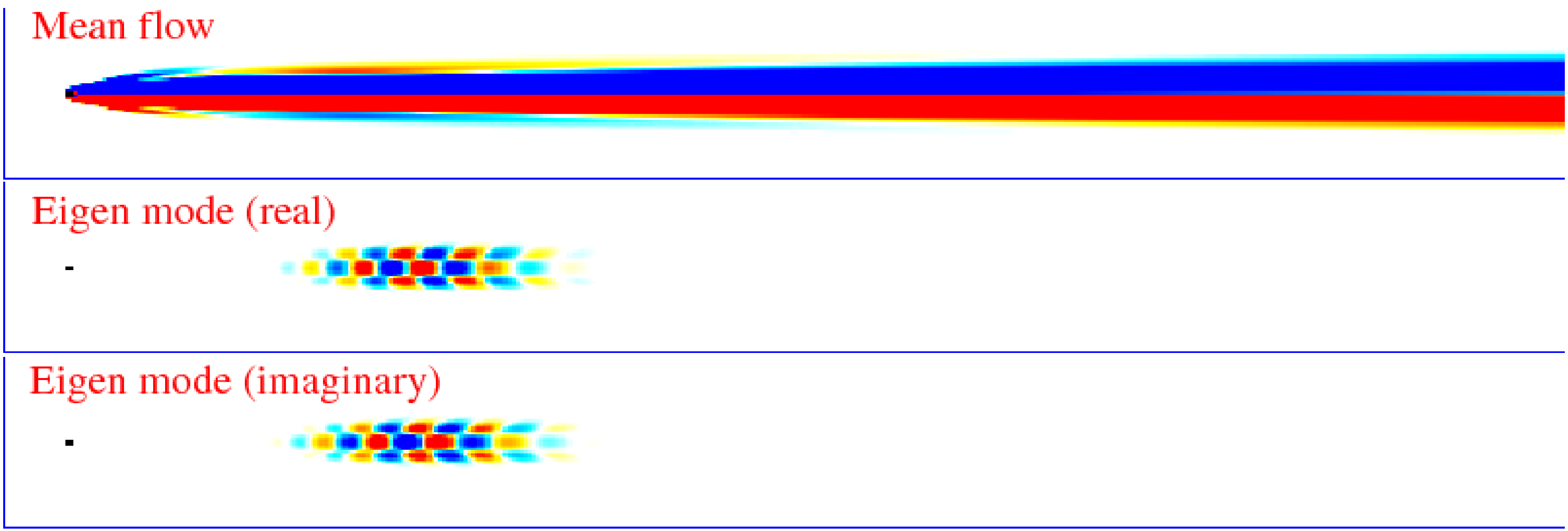
Kumar & Mittal, JFM (2012)



Linear Stability Analysis: Re=150, Time-averaged flows

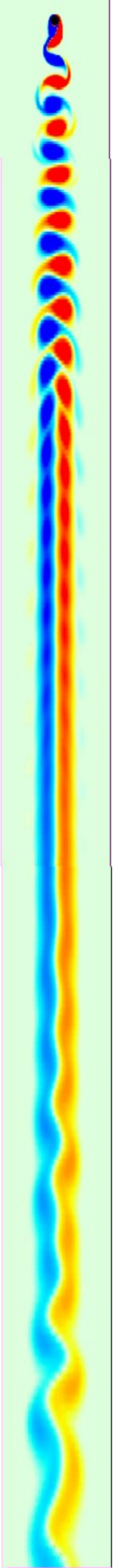
An unstable convective mode, at c=0.8, appears similar to the secondary disturbances

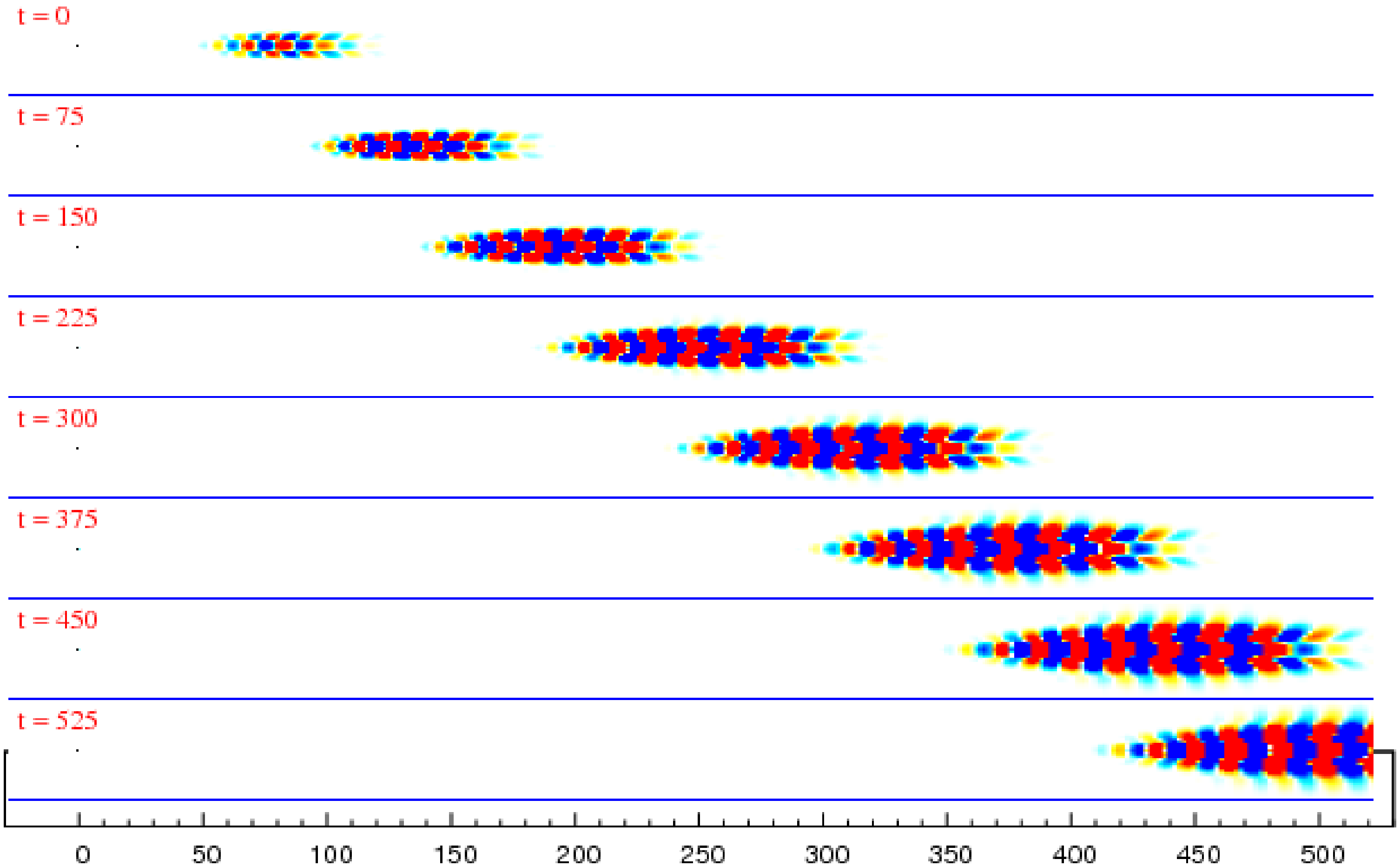
$$\lambda_r = 0.02634,\ \lambda_i = 0.02071,\ St = 0.006591$$

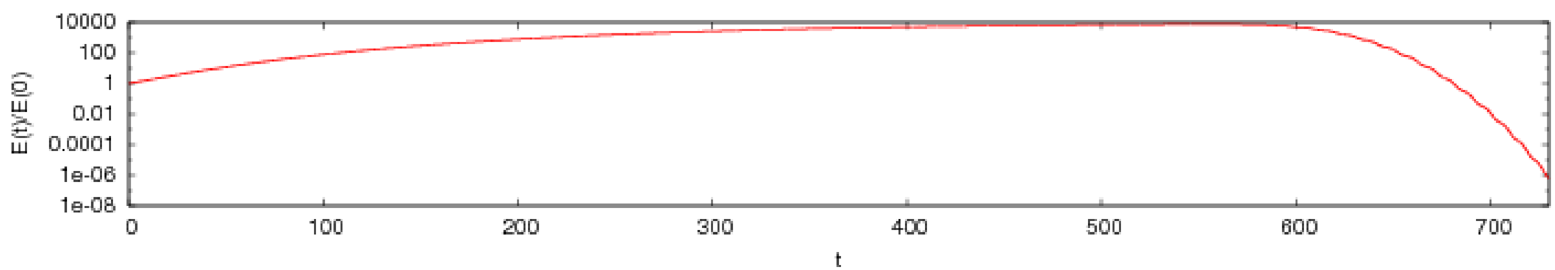


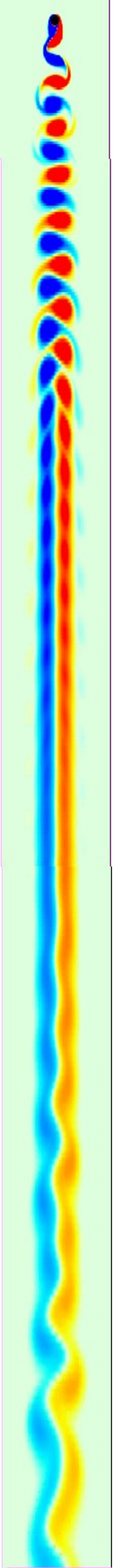
Kumar & Mittal, JFM (2012)

Linear DNS: starting from the unstable convective mode at Re = 150









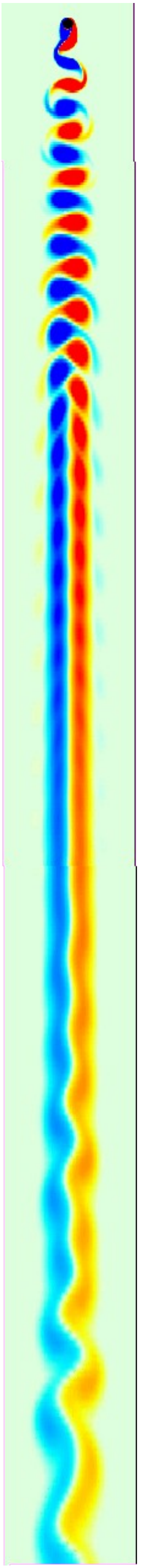
Secondary instability in the far wake:

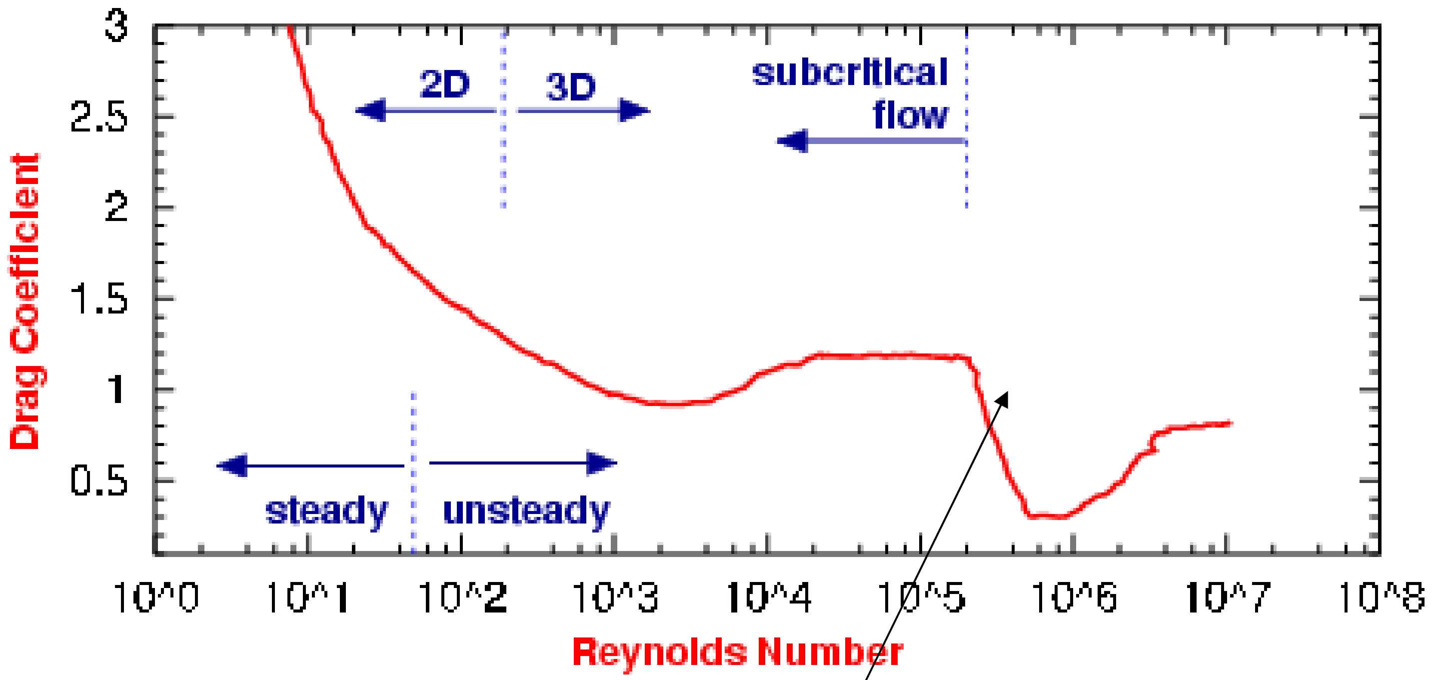
Wave packets are generated due to the convective instability of the time averaged wake

Nonlinear interactions are important in modifying the base profile.

Kumar & Mittal, JFM (2012)

Drag Crisis: Role of shear layer instability

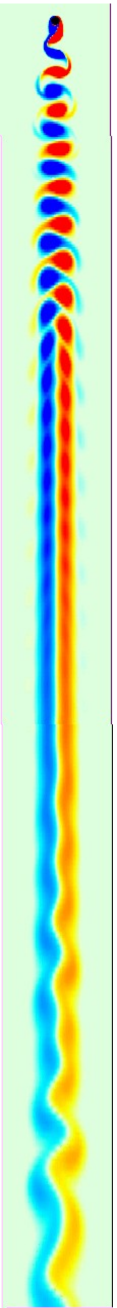


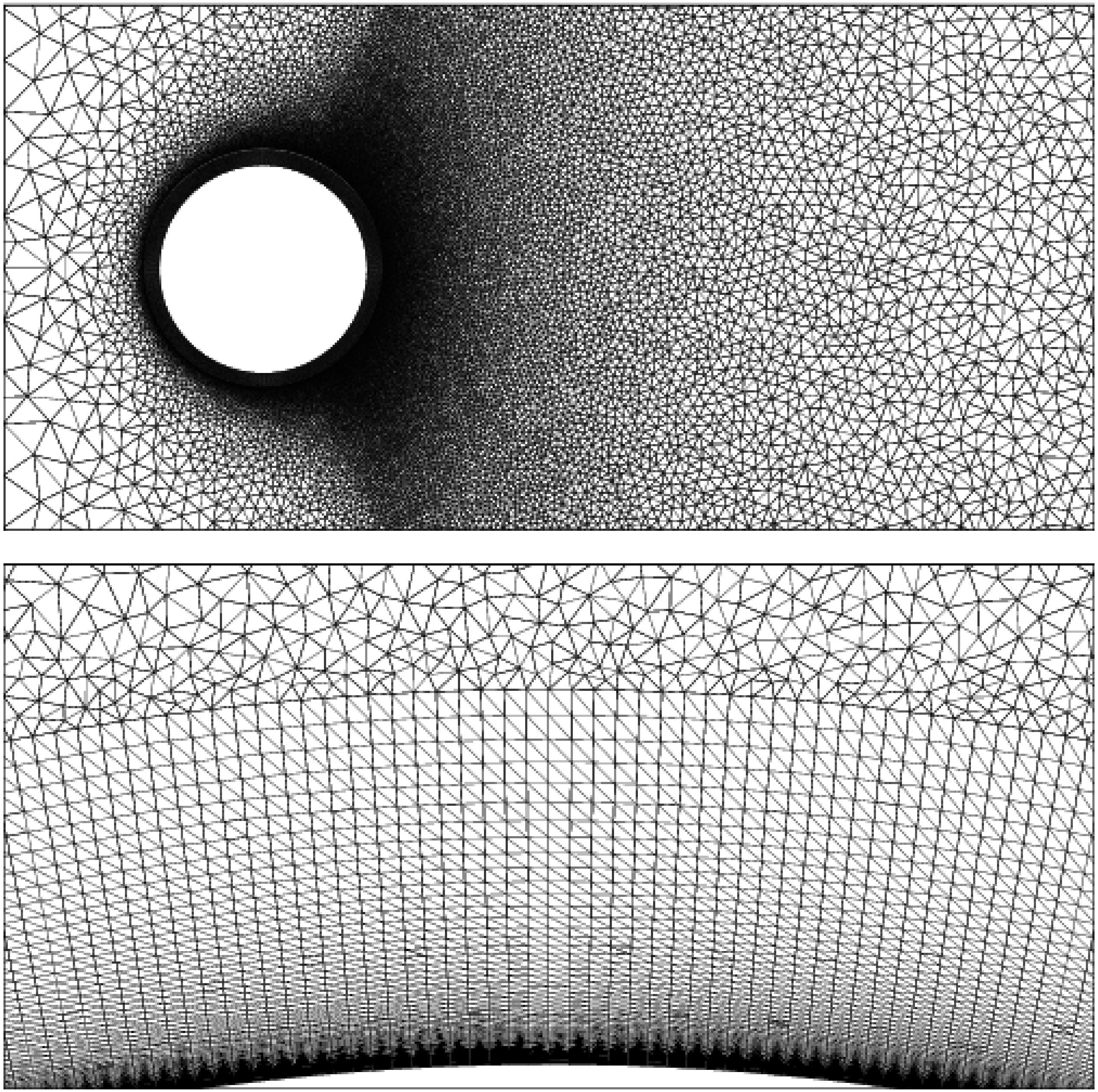


Drag Crisis: Sudden loss in the drag coefficient.
Transition of boundary layer from laminar to turbulent

Question: What is the mechanism of this transition

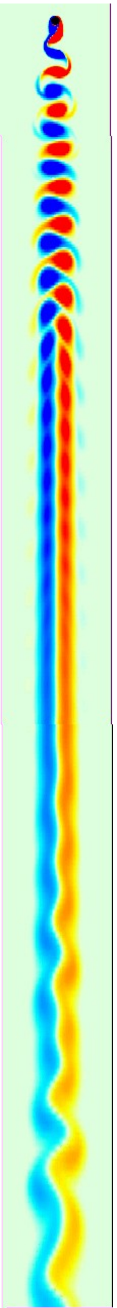
Flow past a stationary cylinder: shear layer instability

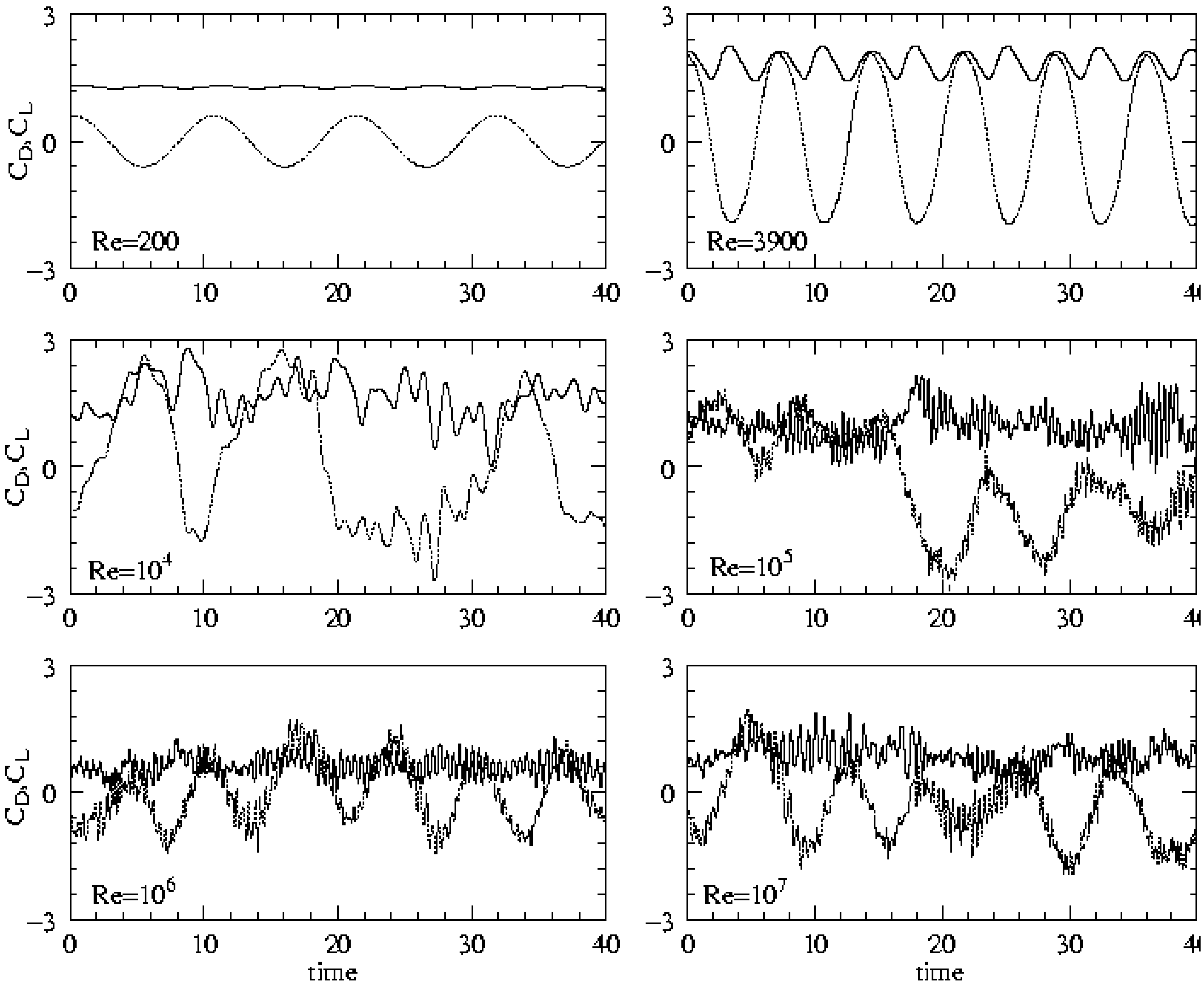




Singh & Mittal, IJNMF (2005)

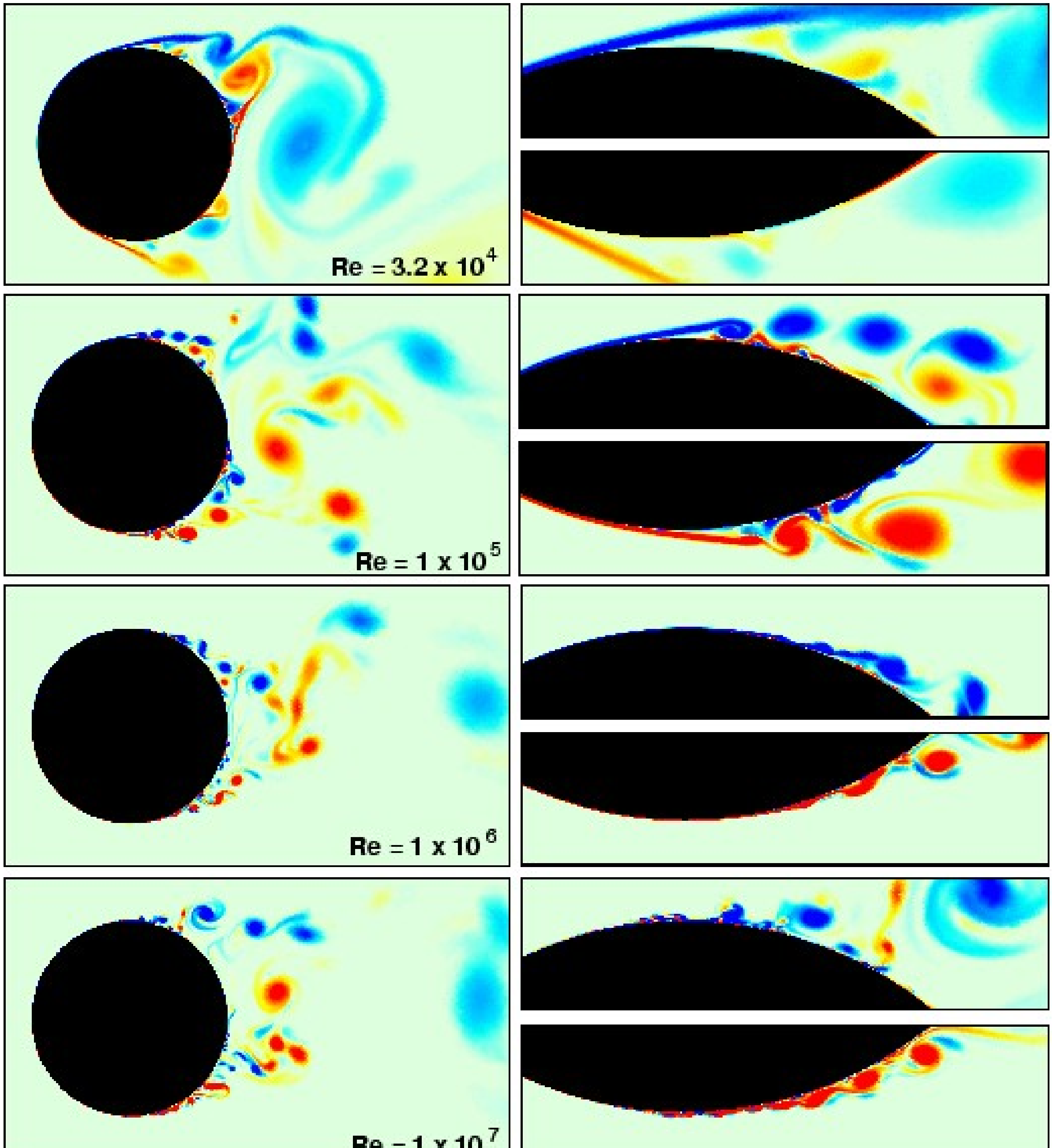
Flow past a stationary cylinder: shear layer instability





Singh & Mittal, IJNMF (2005)

Flow past a stationary cylinder: shear layer instability

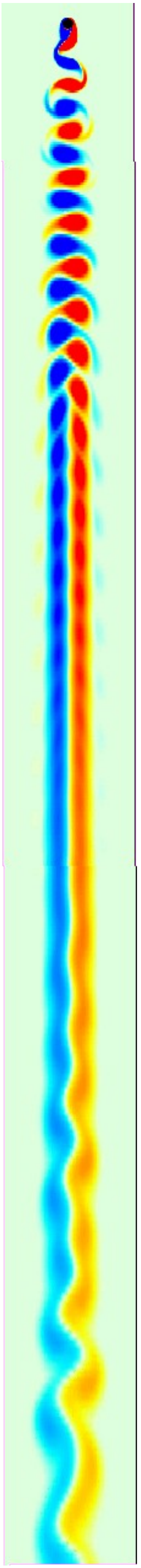


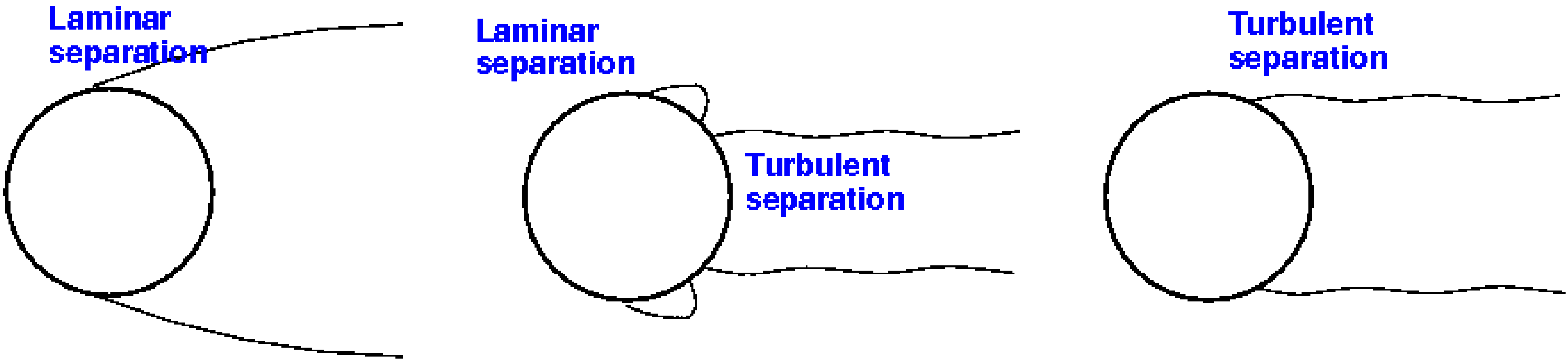
the onset of shear layer instability moves upstream with Re

At the critical Re, the shear layer vortices cause mixing of flow in the boundary layer

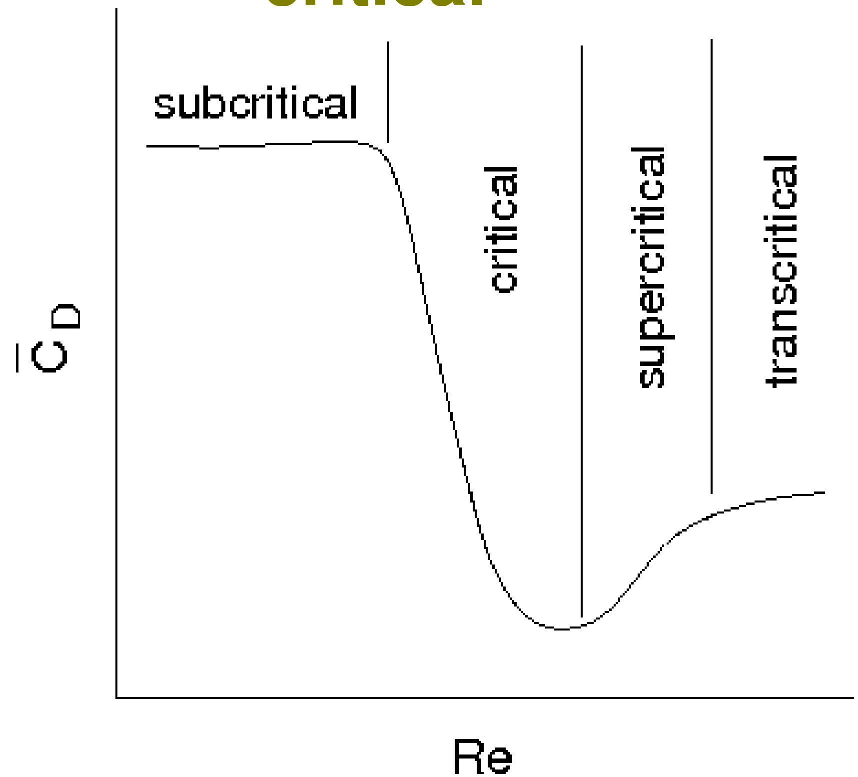
Singh & Mittal, IJNMF (2005)

Laminar Separation Bubble

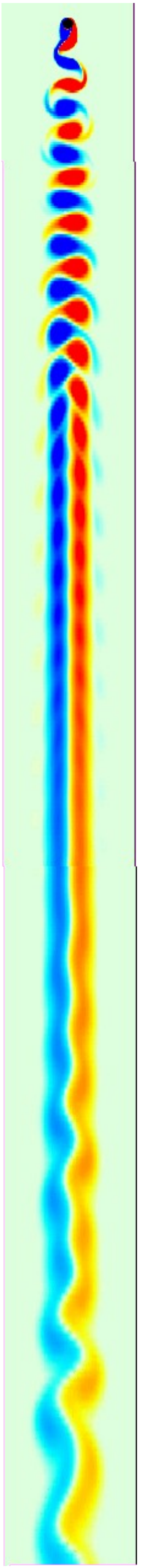


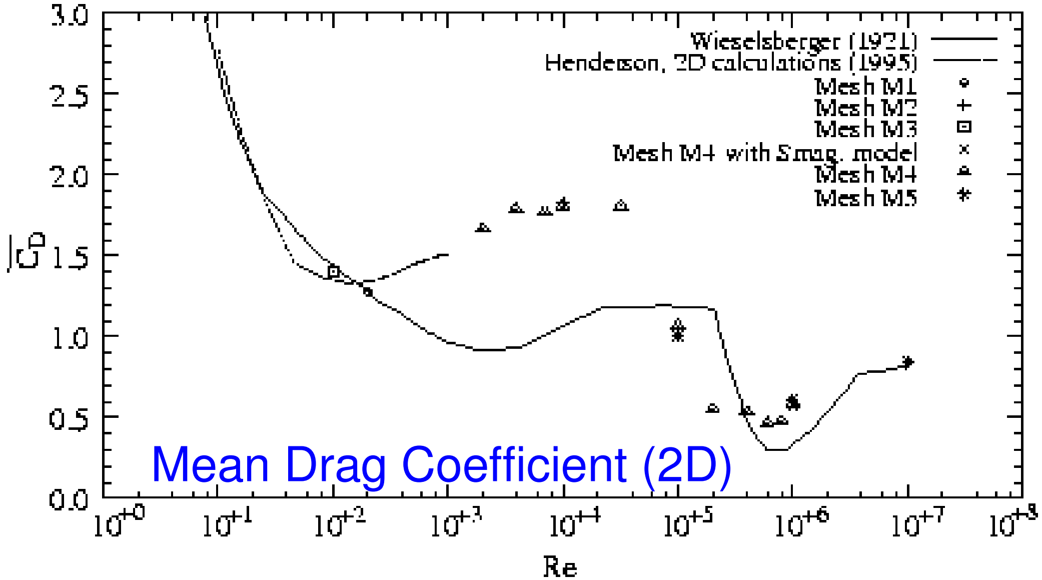


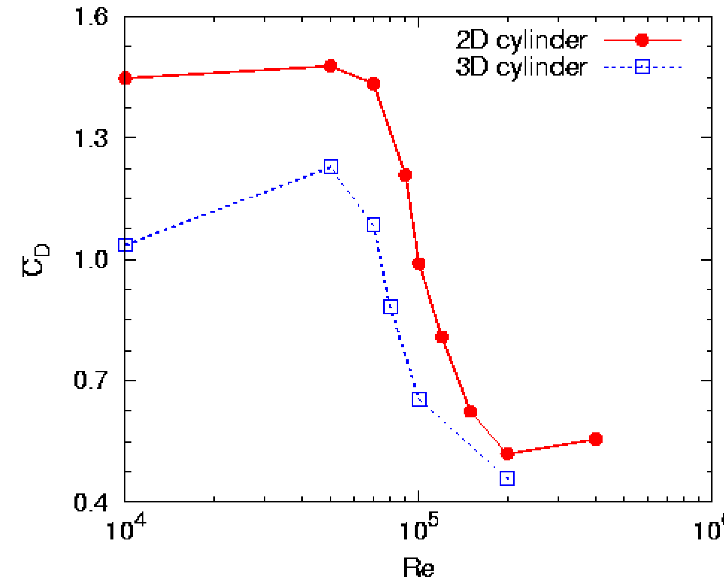
sub-critical | critical | super-critical

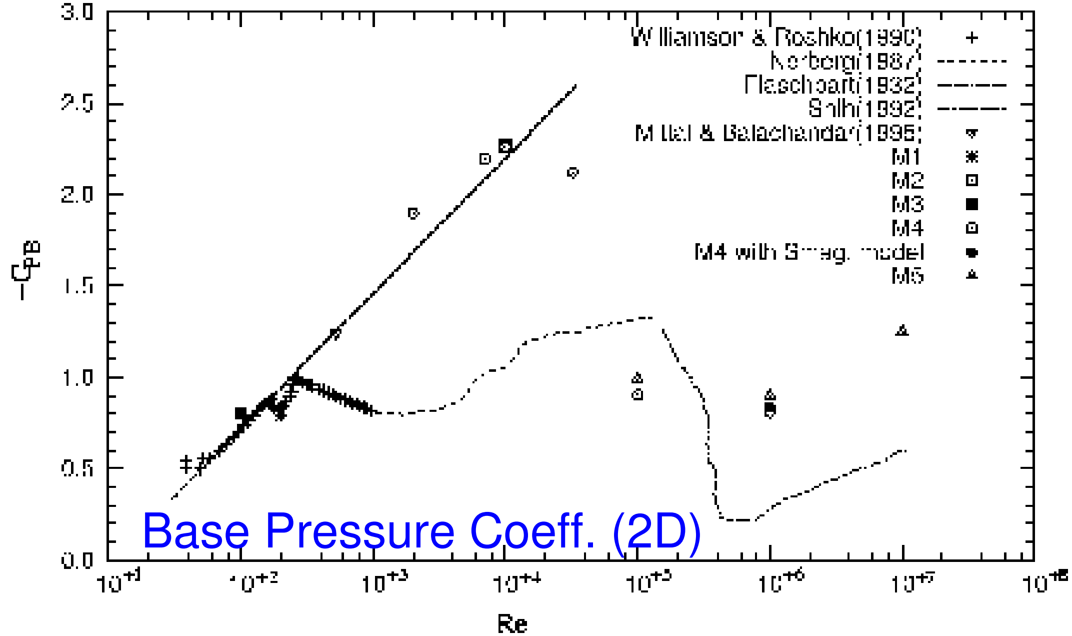


Drag Crisis: time-averaged coefficients





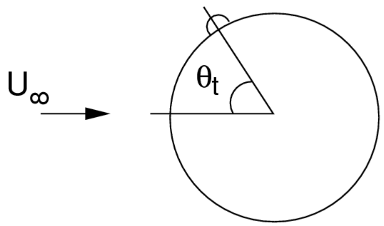


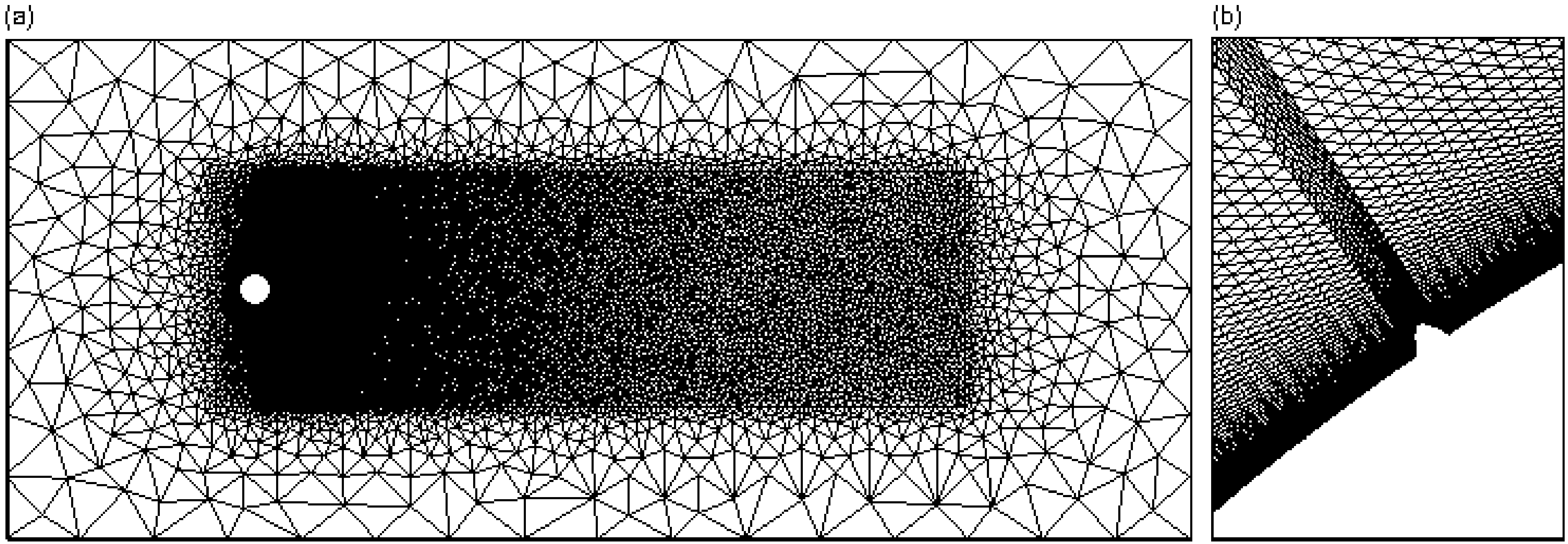


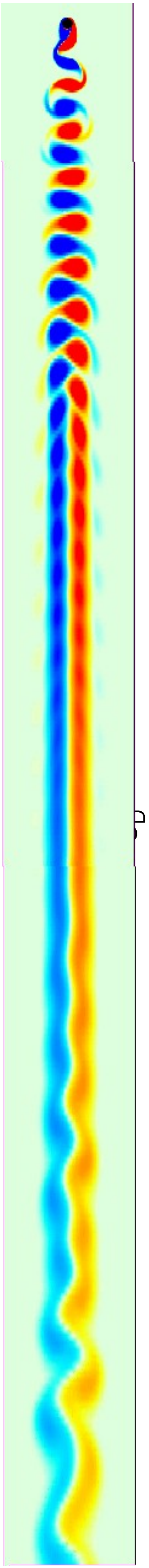
Singh & Mittal, IJNMF (2005)

Behara & Mittal JFS (2011)

Drag Crisis: Cylinder with roughness element

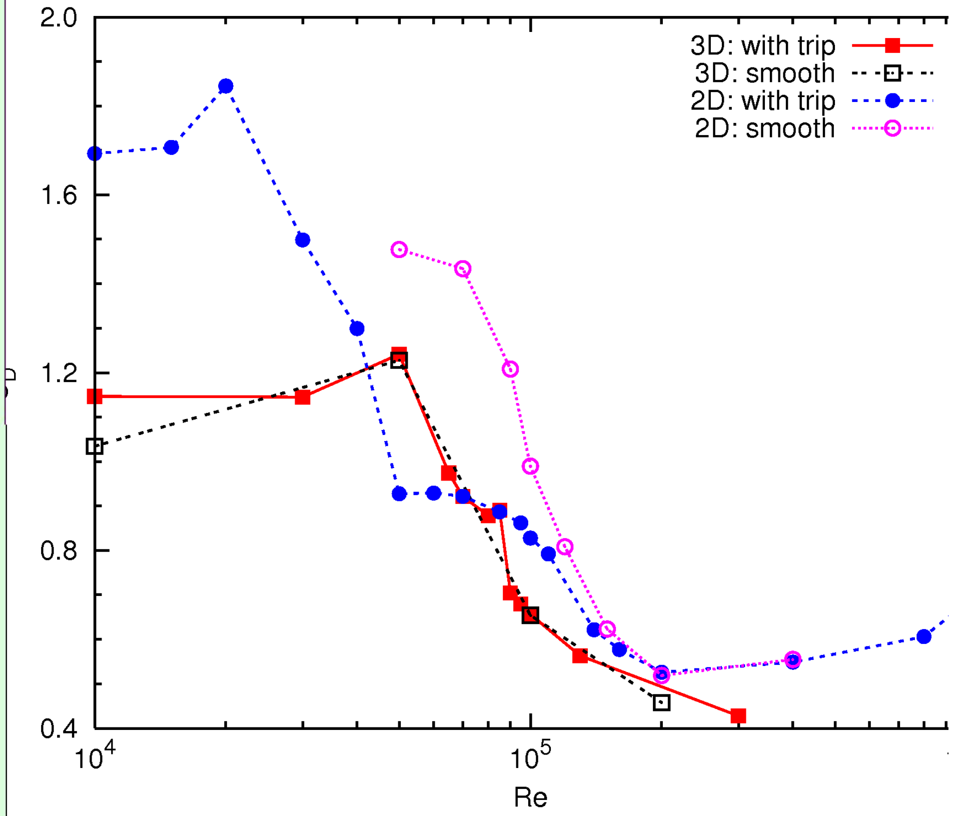




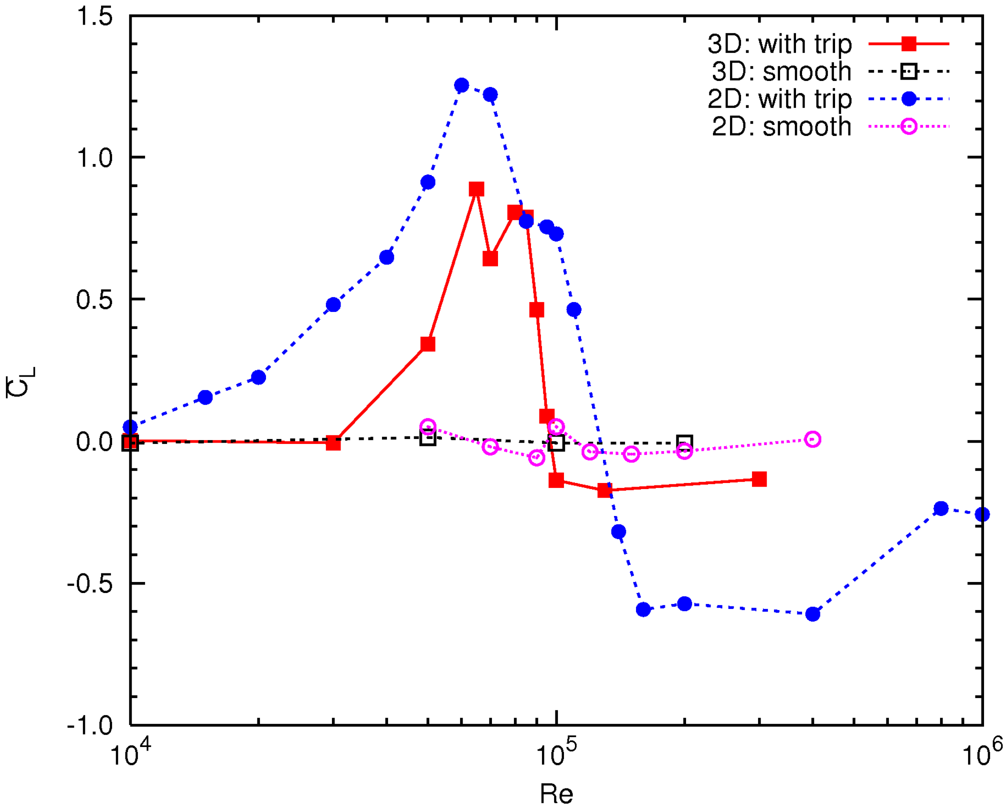


Effect of a roughness element (trip)

Drag coefficient

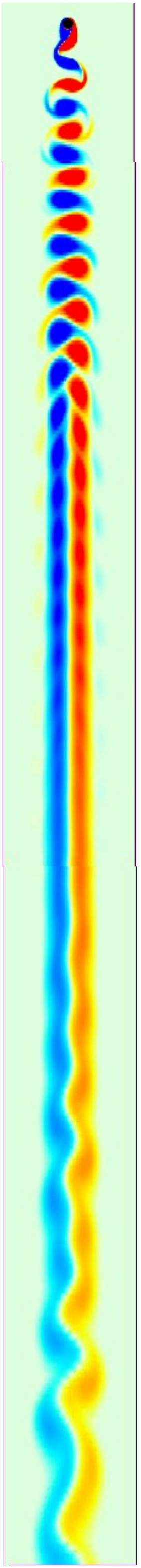


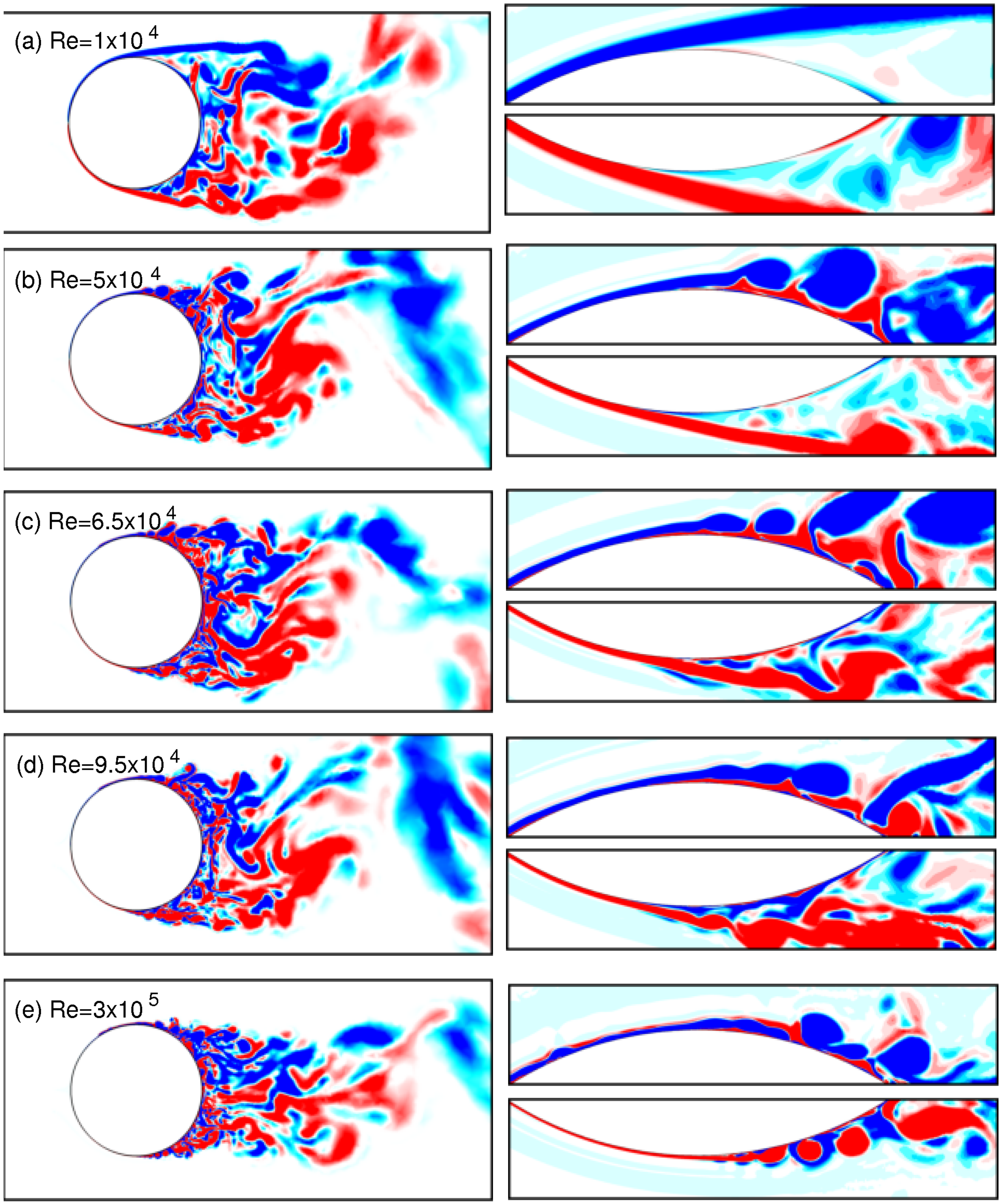
Lift coefficient



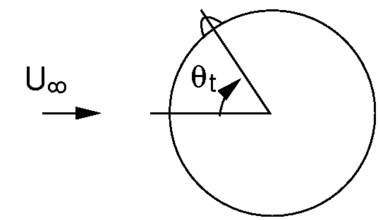
Behara & Mittal, JFS (2011)

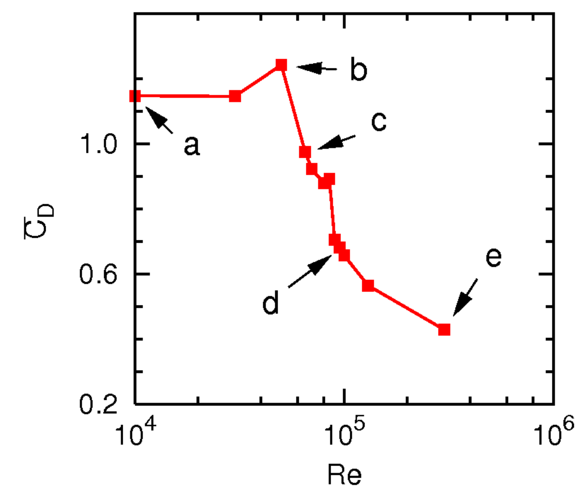
Cylinder with trip: instantenous vorticity

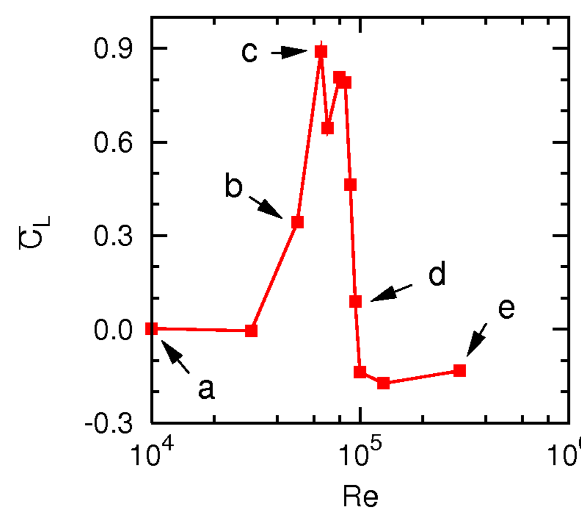




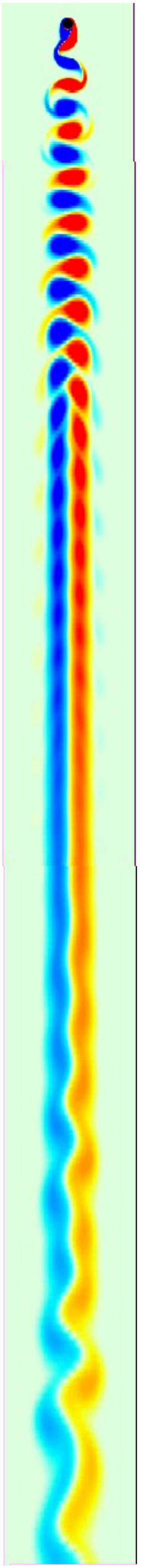
Behara & Mittal
JFS (2011)

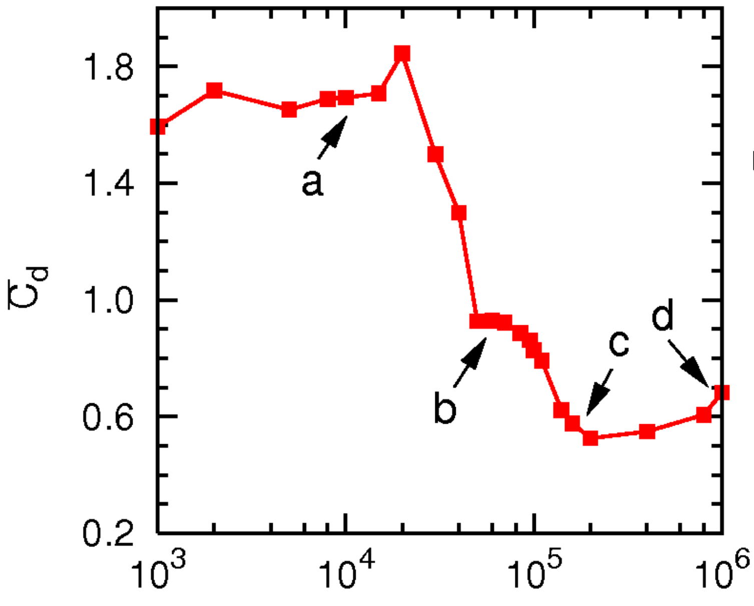


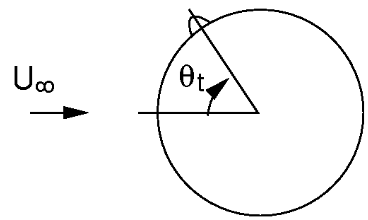


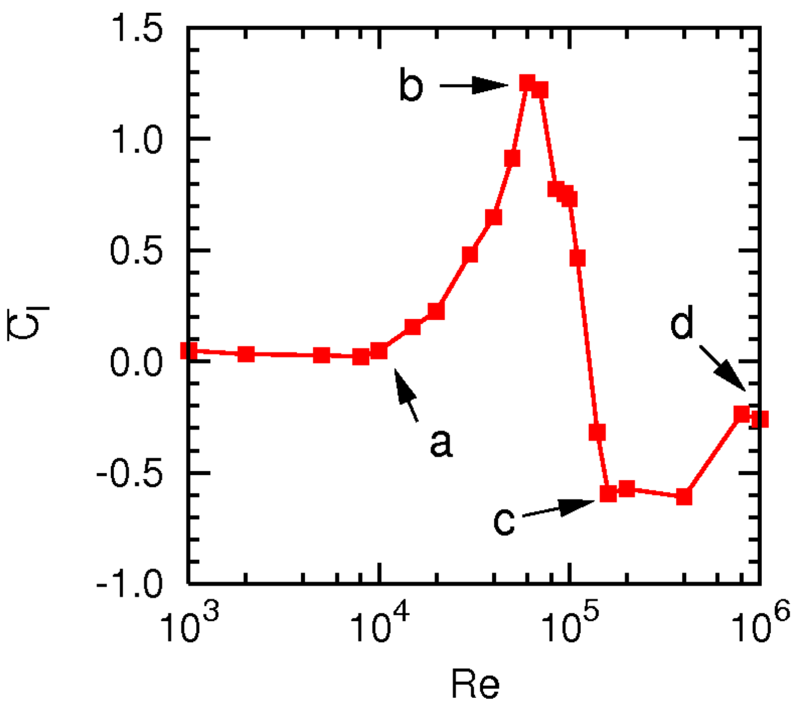


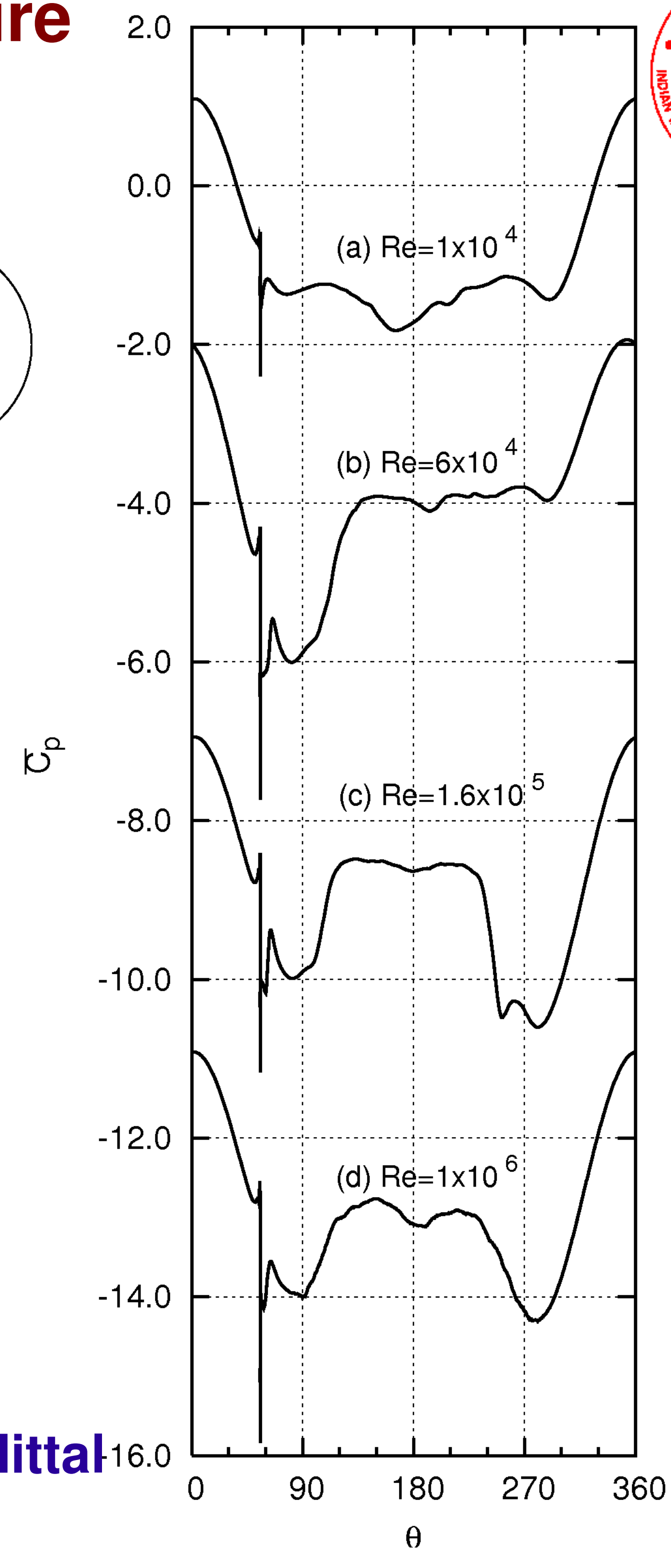
Cylinder with trip: surface pressure





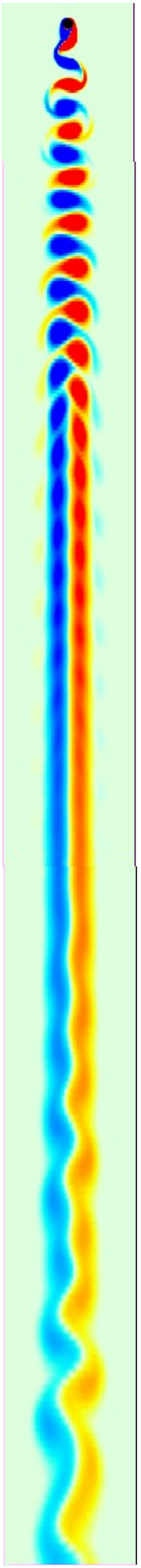


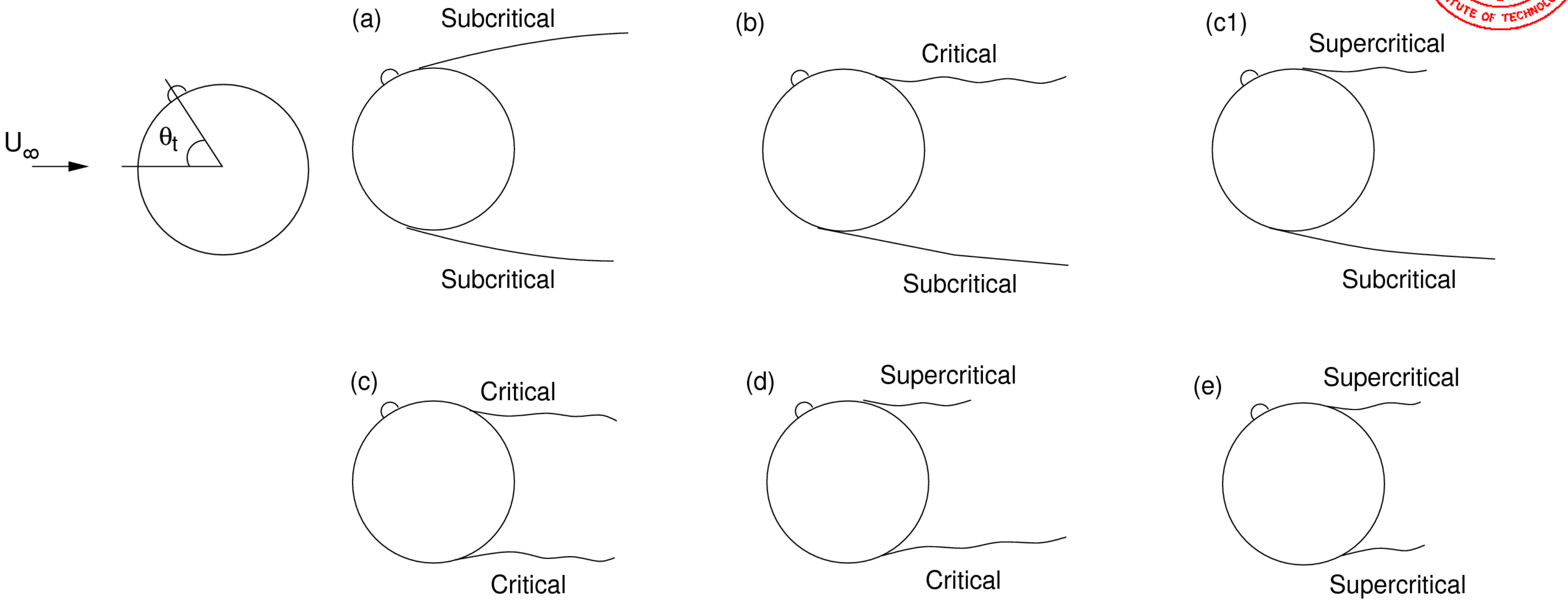


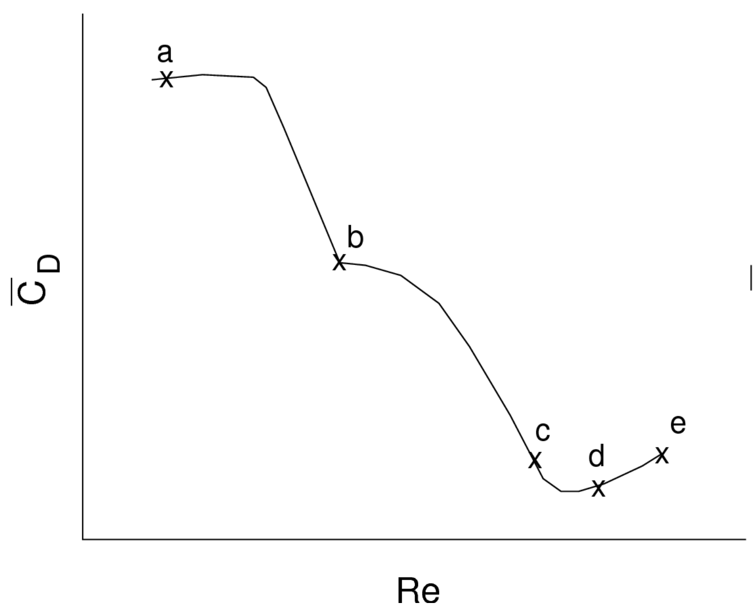


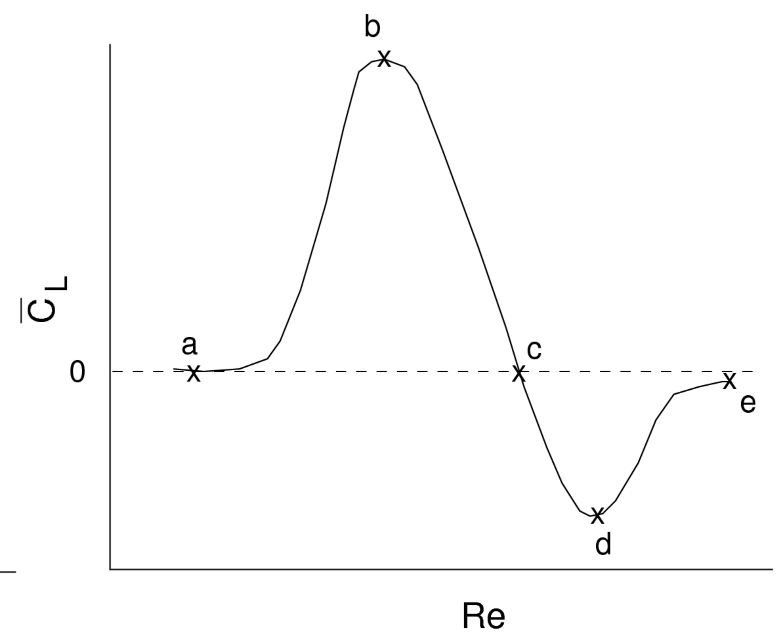
Behara & Mittal
JFS (2011)

Drag Crisis: in the presence of a trip



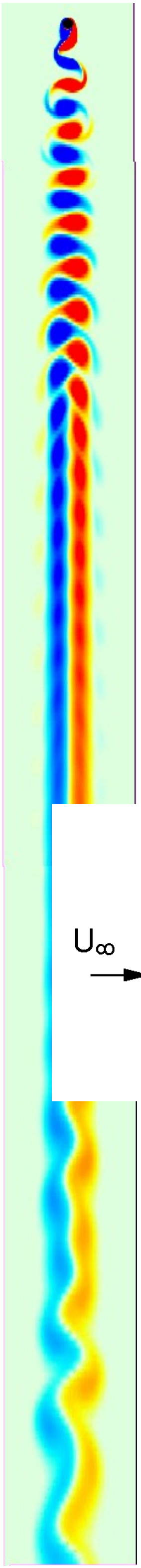






Behara & Mittal
JFS (2011)

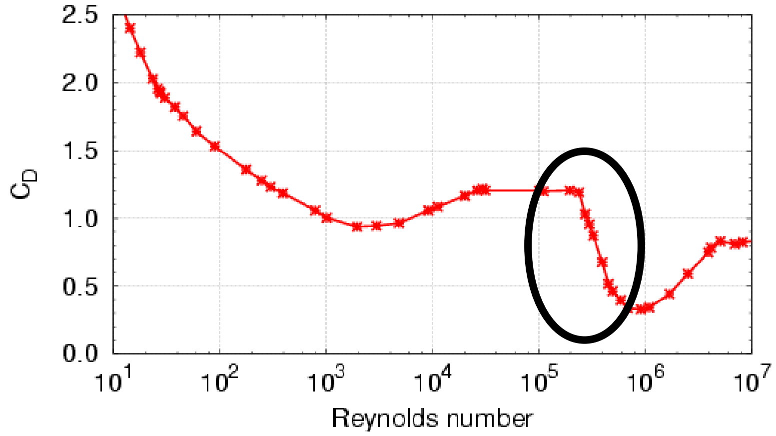
Cylinder v/s sphere

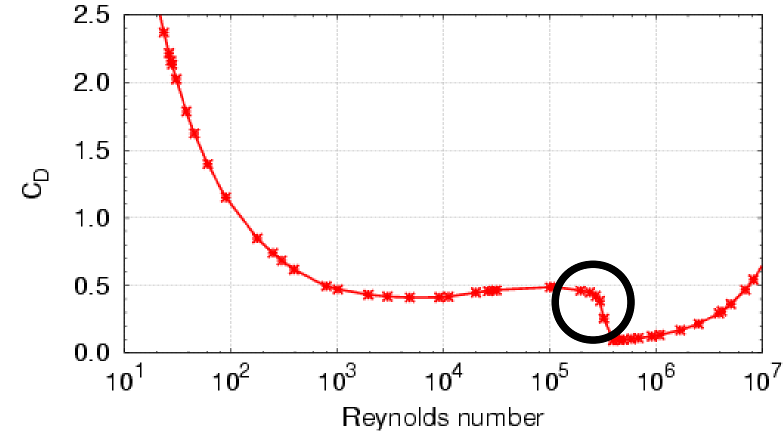


Cylinder

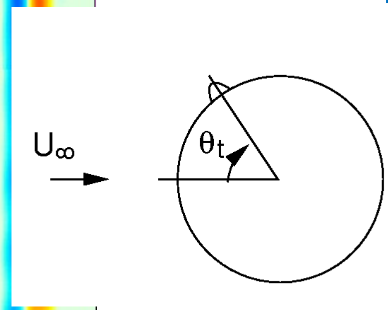
Sphere

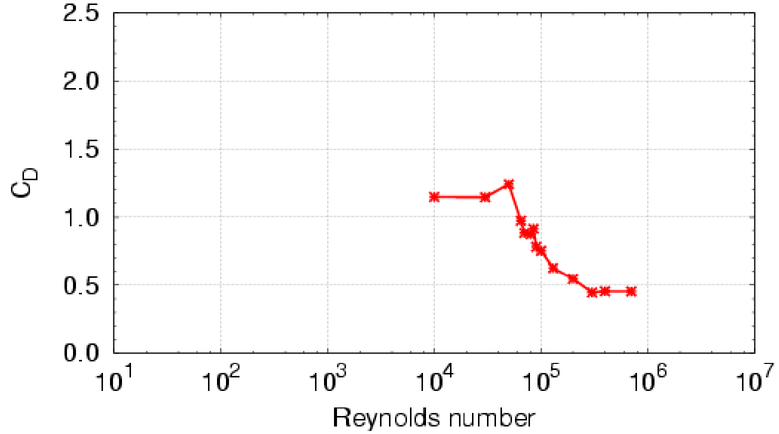
Smooth

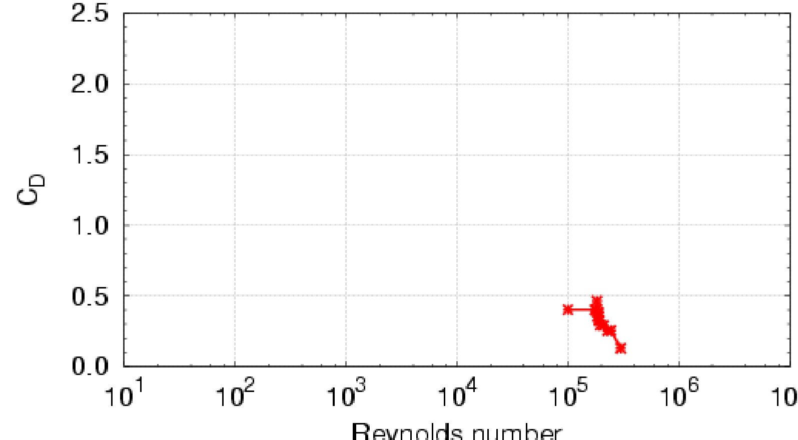


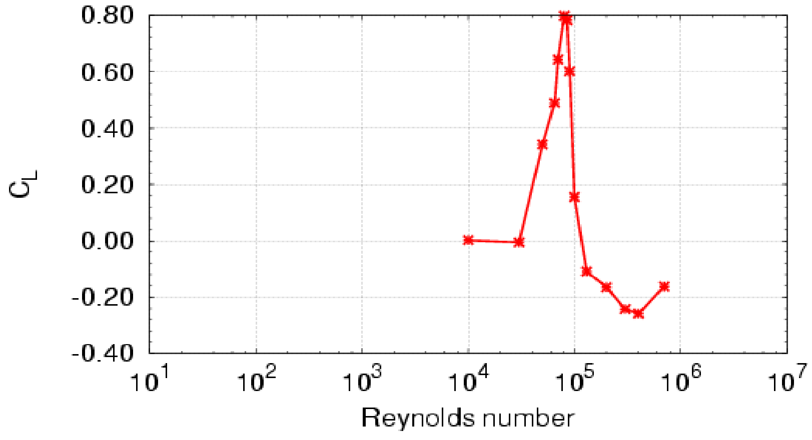


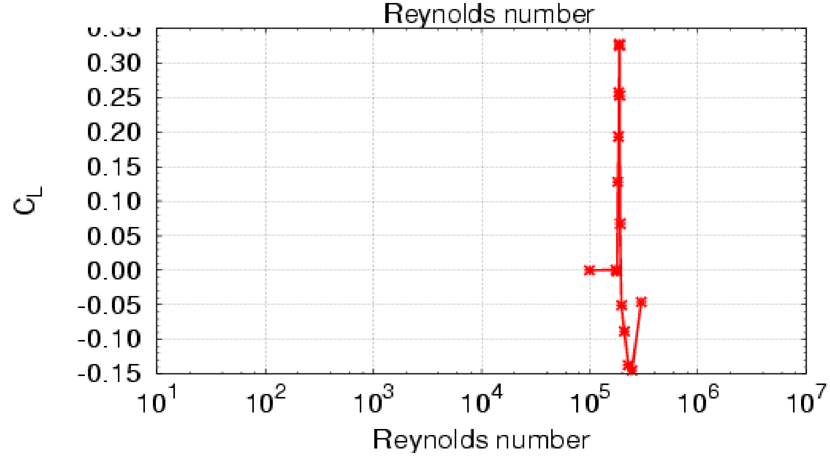
With trip



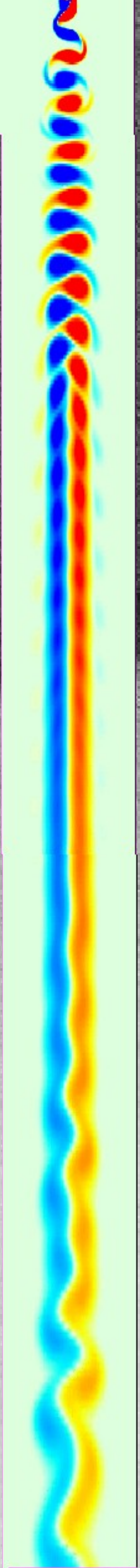








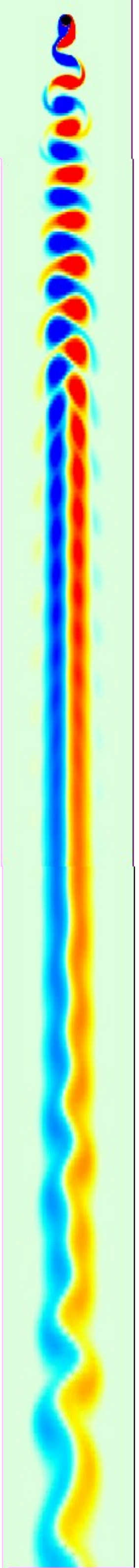
Aerodynamic Analysis of Shuttlecocks



Types of Shuttlecocks

Duck Feather

Synthetic



Aerodynamics of Shuttlecocks

Very little known – mostly experiments

Players prefer feather shuttlecock

Feather shuttlecock – brittle, expensive

Need an improved design for a synthetic shuttlecock

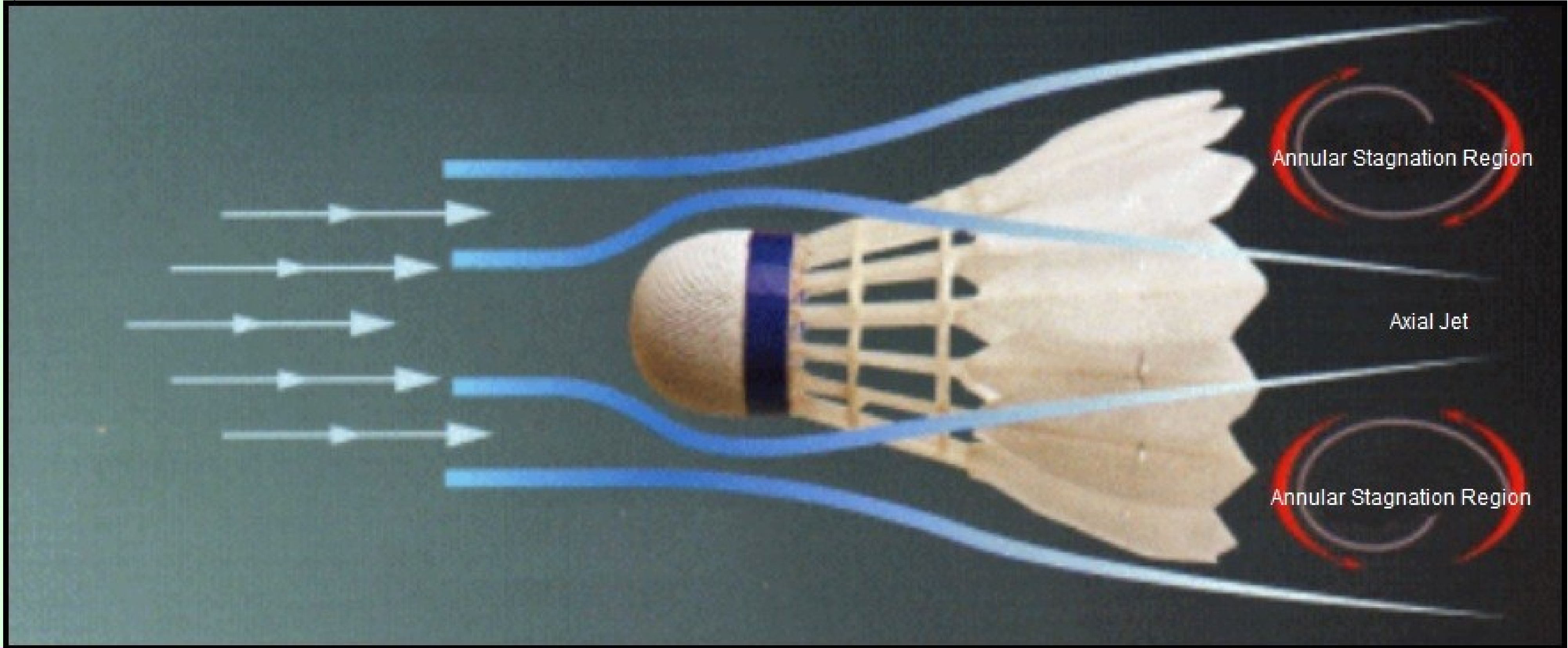
Begin by finding the difference in their aerodynamics

Aerodynamics of Shuttlecocks

Cooke (1996), Engg of sports

Axial jet, Annular stagnation region in wake

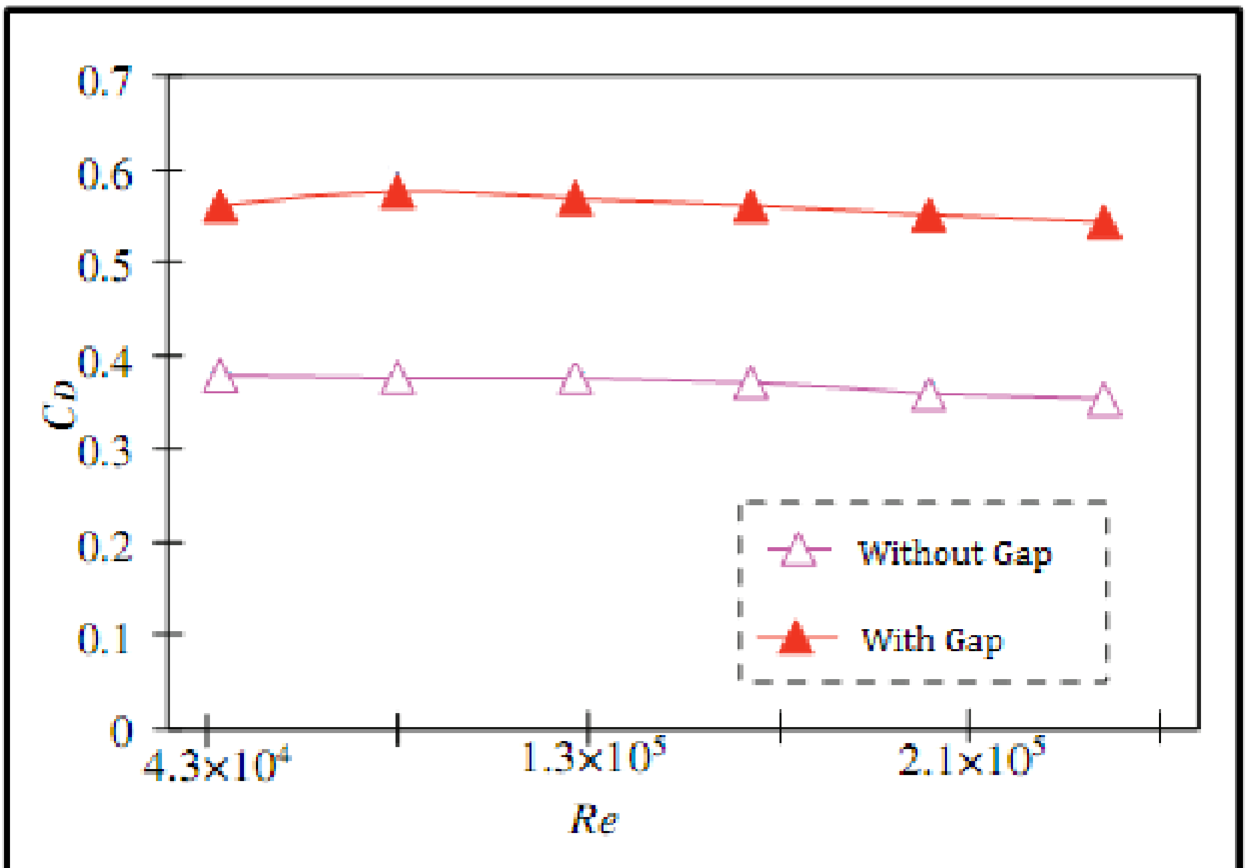
Gap upstream of skirt increases drag

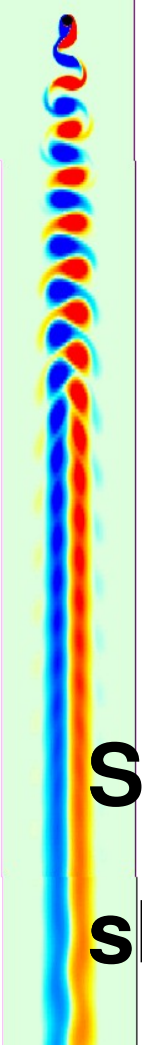


Aerodynamics of Shuttlecocks

Kitta et al. (2011), APCST

Studied a feather stuttlecock, with and without gap

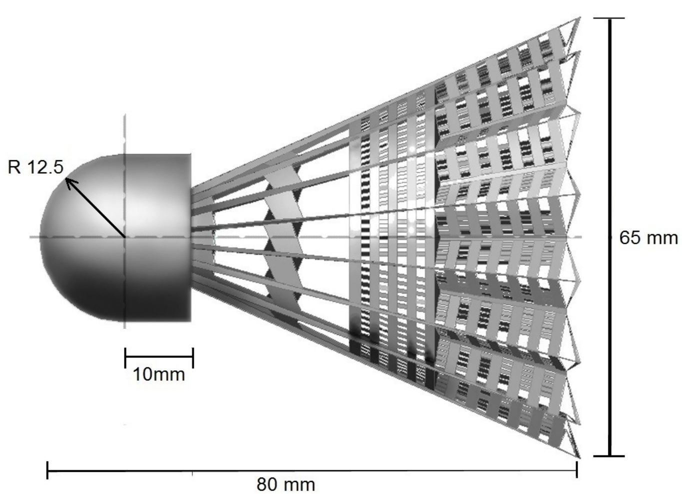




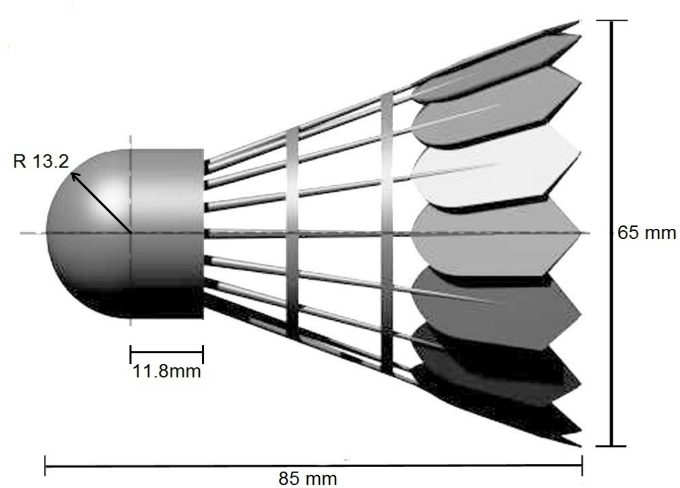
Aerodynamics of Shuttlecocks

3 Models

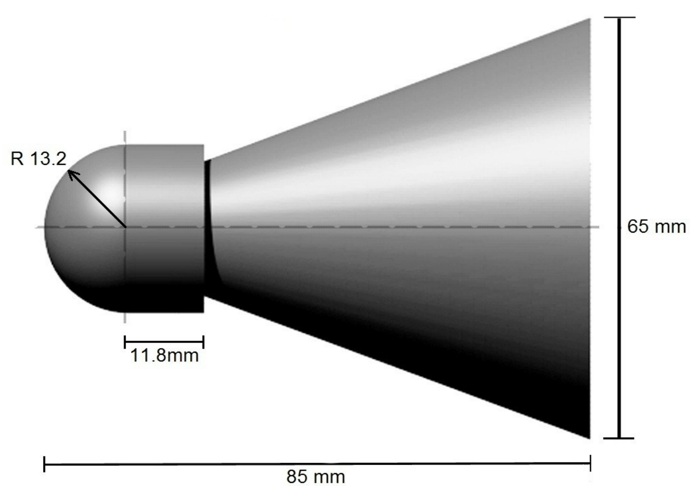
Synthetic shuttlecock



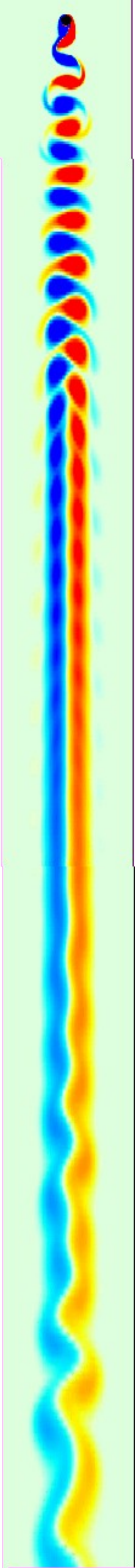
Feather shuttlecock

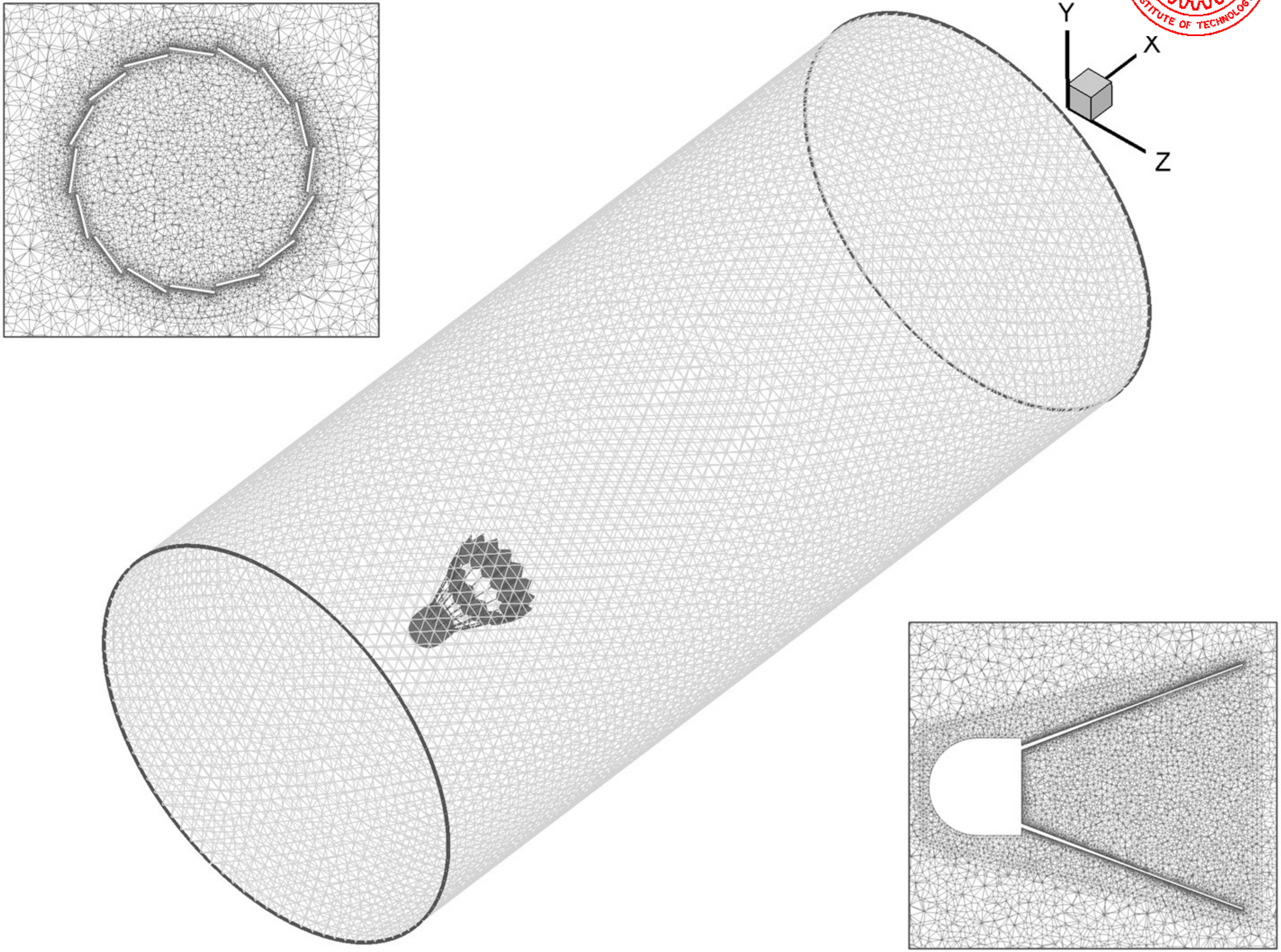


No Gap shuttlecock

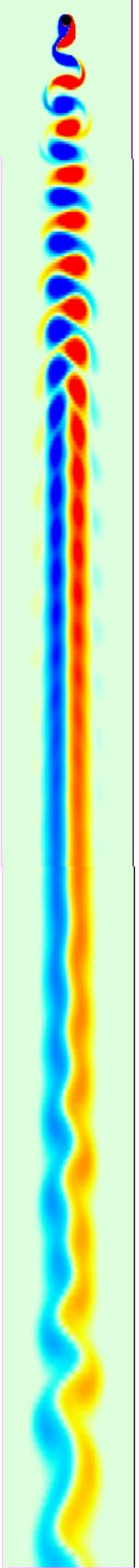


Mesh for Shuttlecock



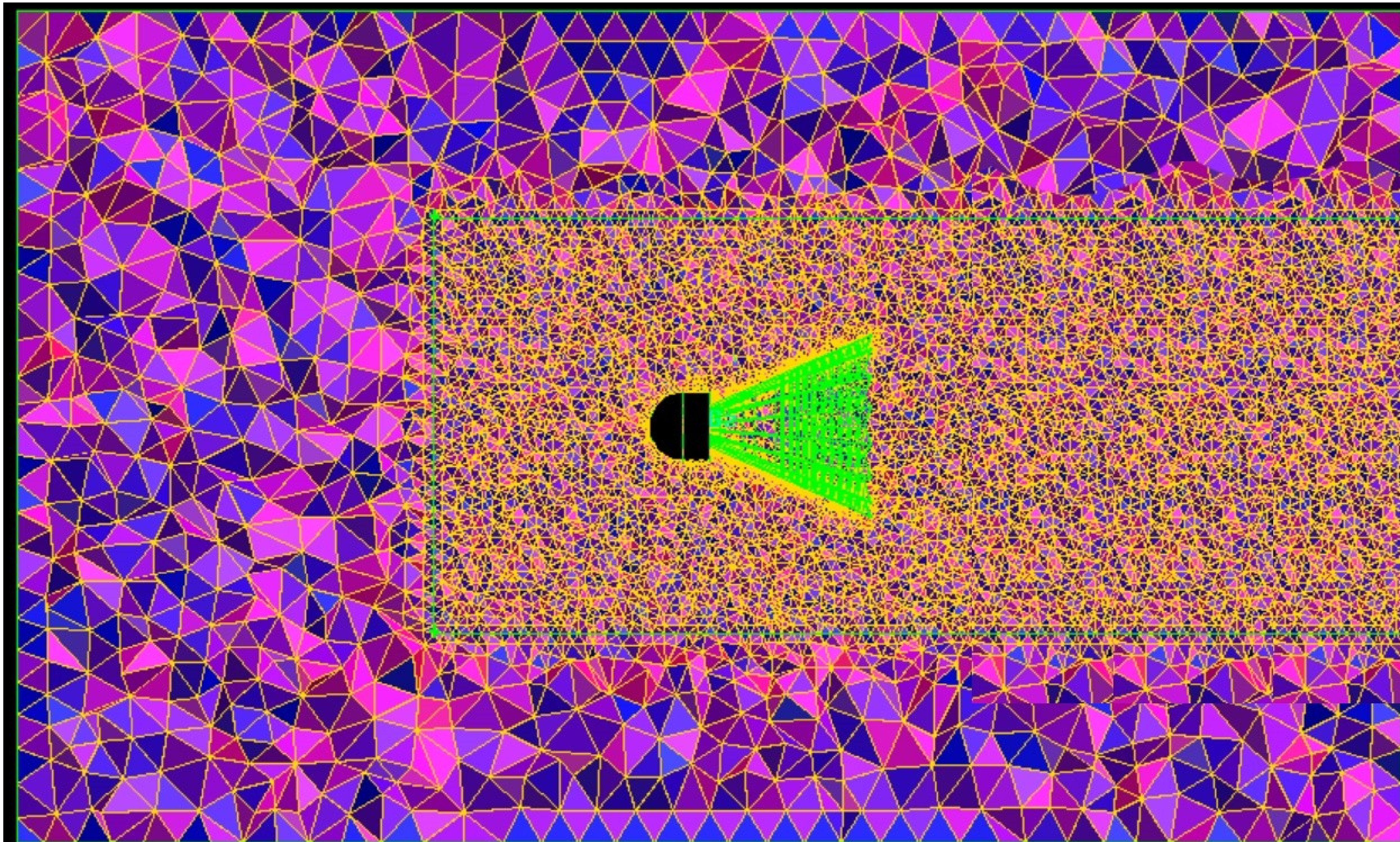


~ 3.2 million tetrahedral elements



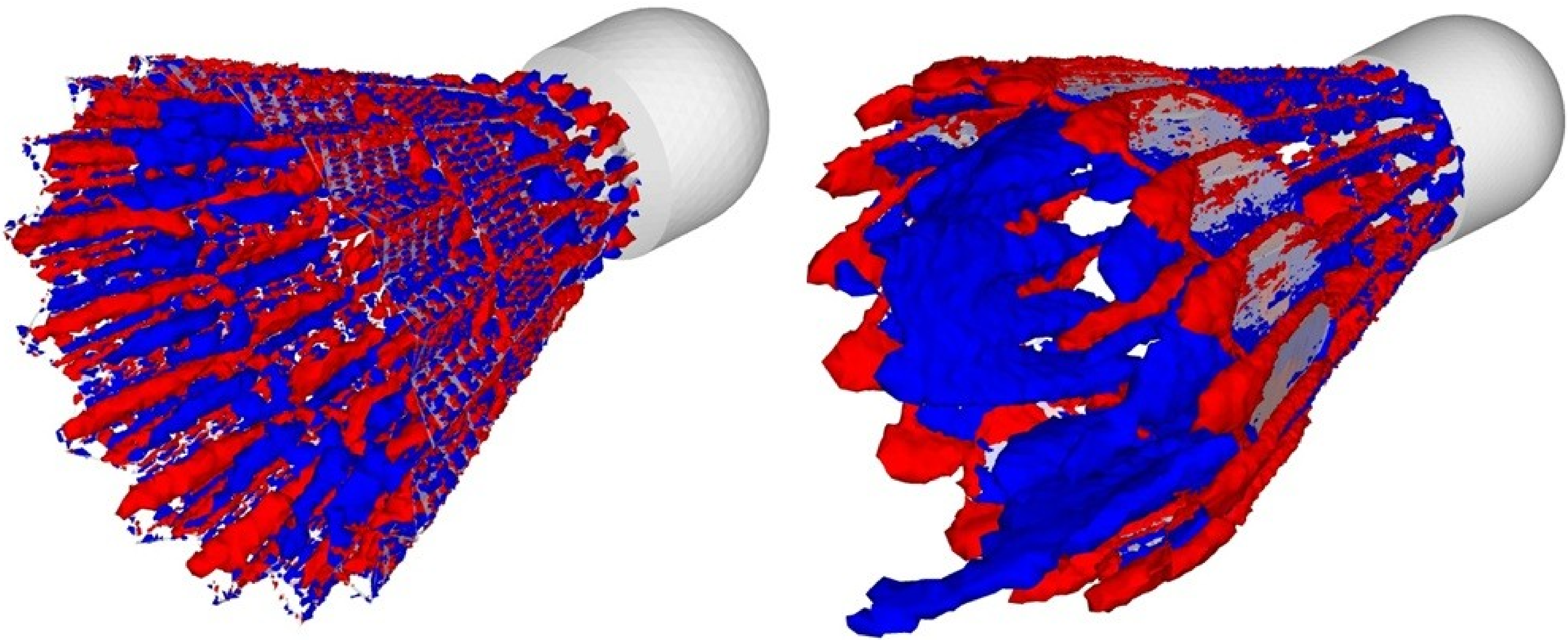
Synthetic Shuttlecock

Typical mesh

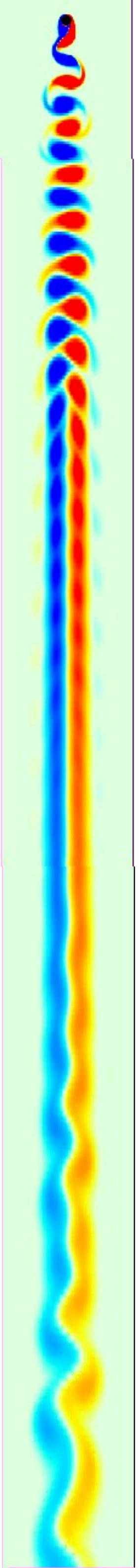


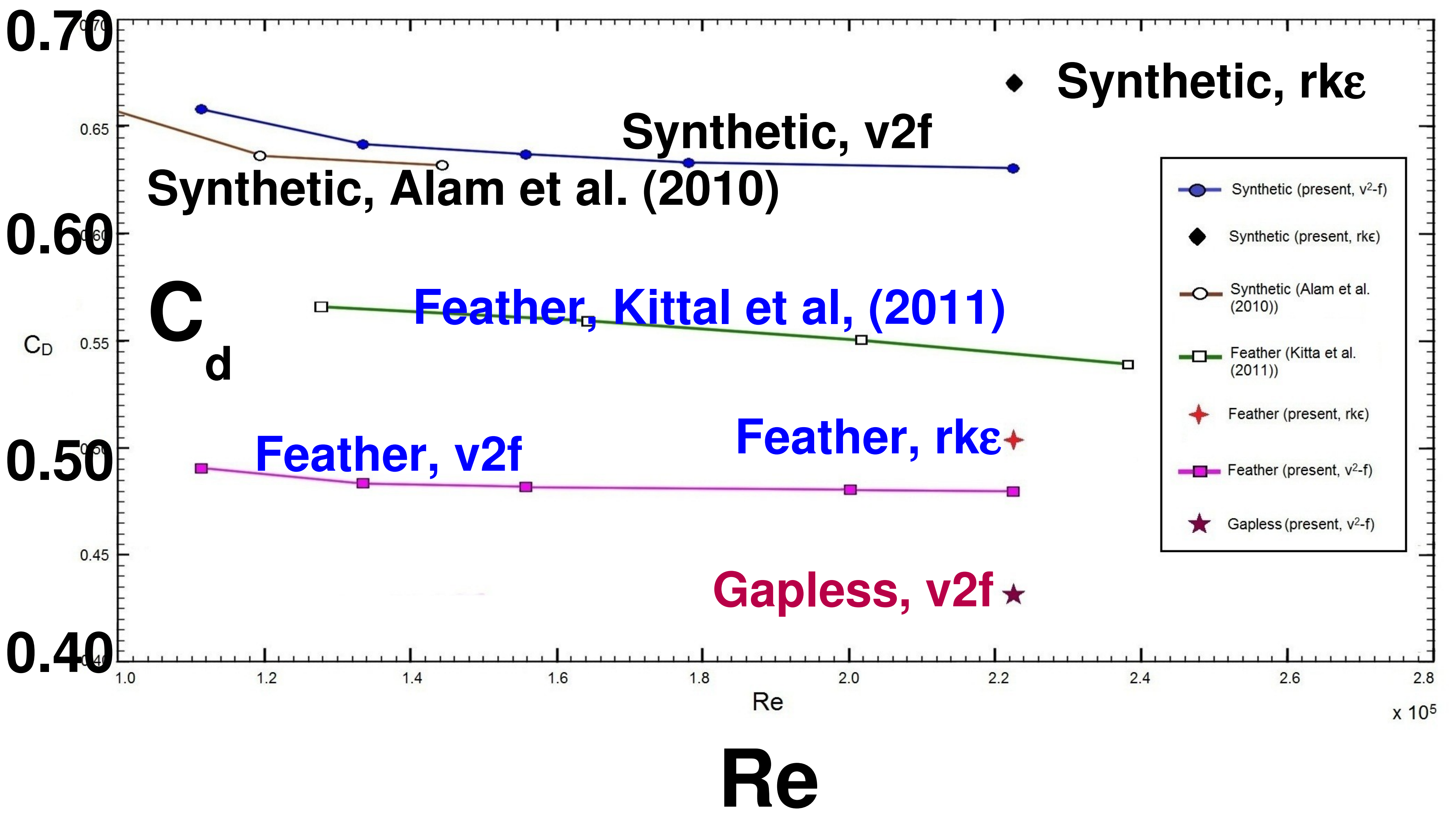
Streamwise vorticity

U=50 m/s, Re = 2.22×10^5

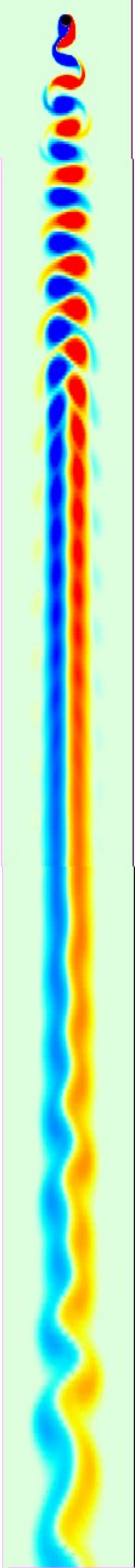


Drag Coefficient: Shuttlecock



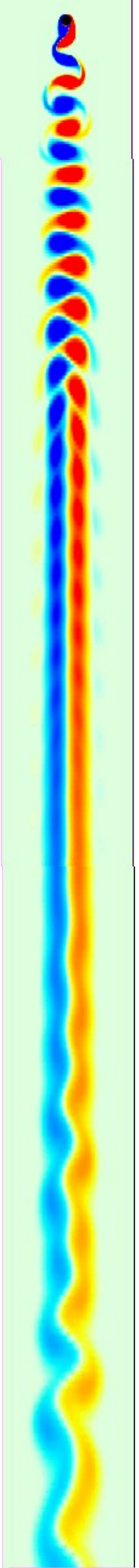


Drag Coefficient: Shuttlecock



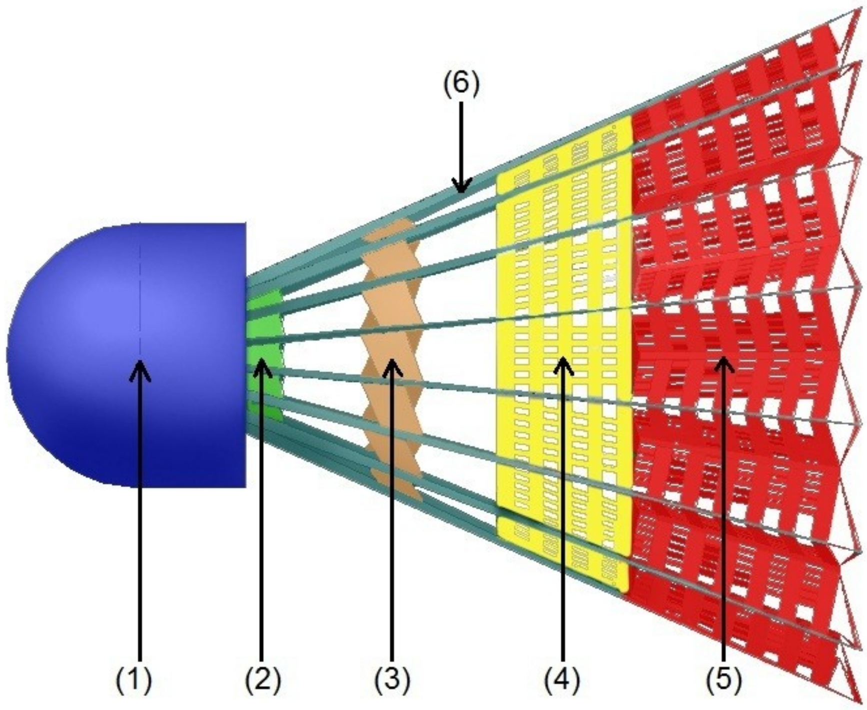
U=50 m/s, Re = 2.22 X 10^5

	Mesh 1 3.2 million elements	Mesh 2 5.6 million elements
Synthetic	0.632	0.668
Feather	0.479	0.480



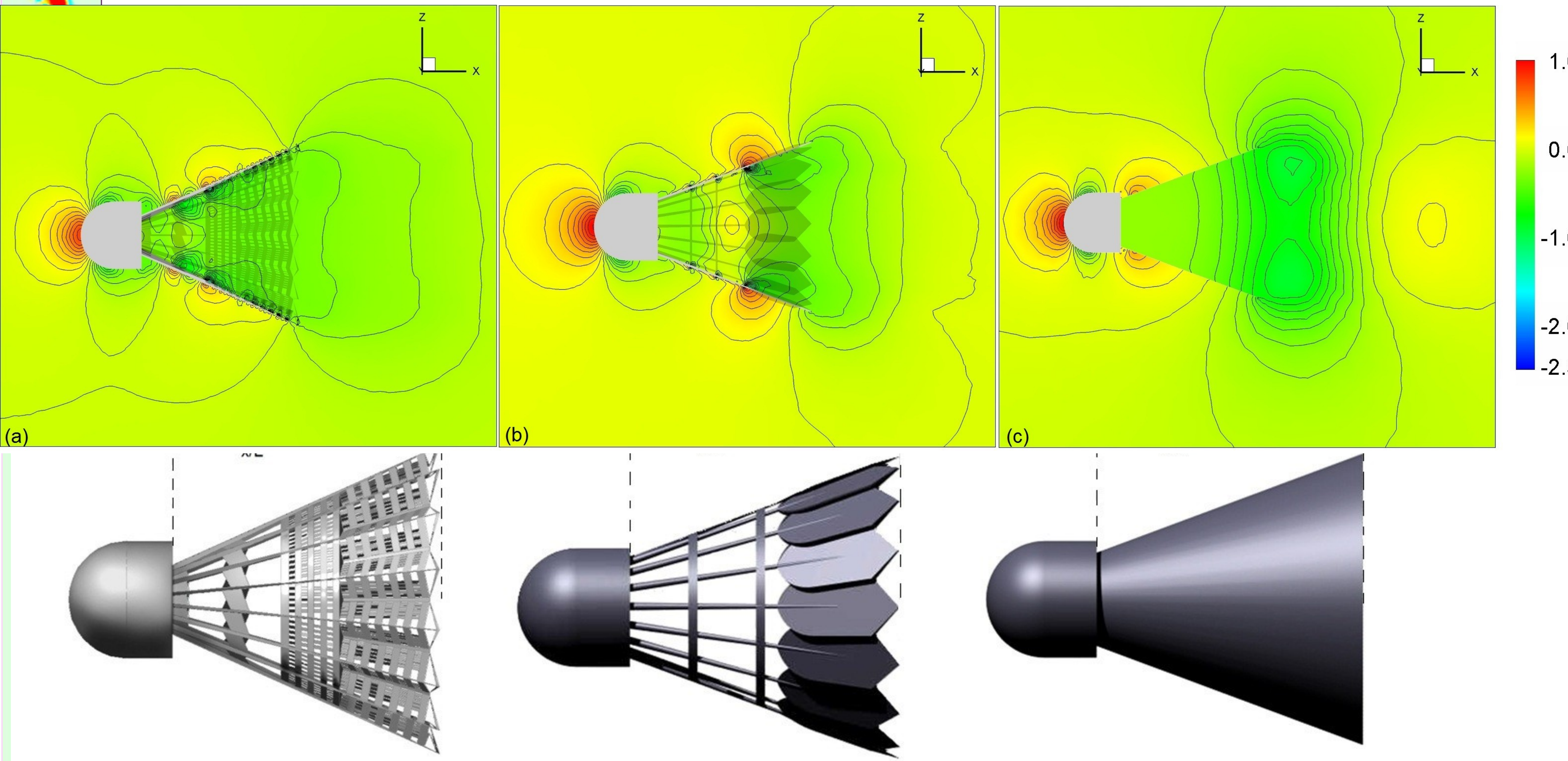
Drag Coefficient: Synthetic Shuttlecock

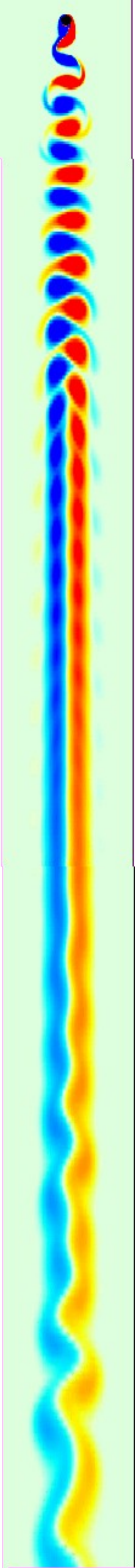
U=50 m/s, Re = 2.22 X 10^5



Region	% C_{DP}	% C_{Dv}
(1)	10.754	0.366
(2)	-0.473	0.011
(3)	8.205	0.034
(4)	34.936	0.146
(5)	39.564	1.015
(6)	5.031	0.404

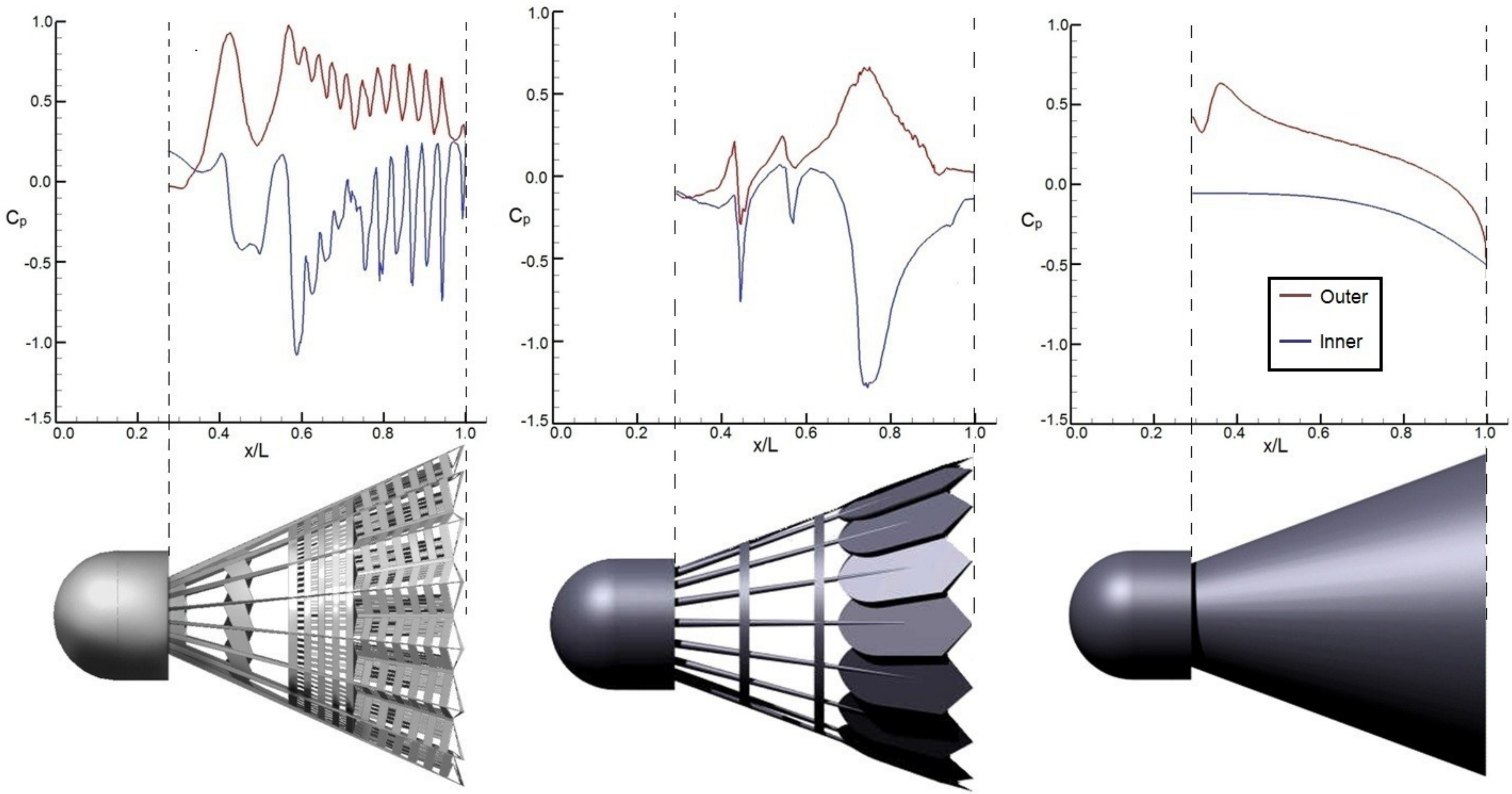
Pressure Coefficient:
U=50 m/s, Re = 2.22 X 10^5

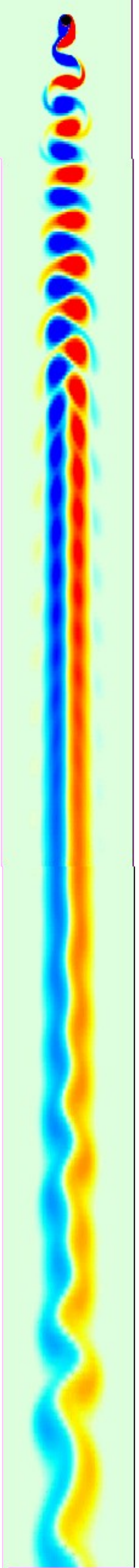




Pressure Coefficient:

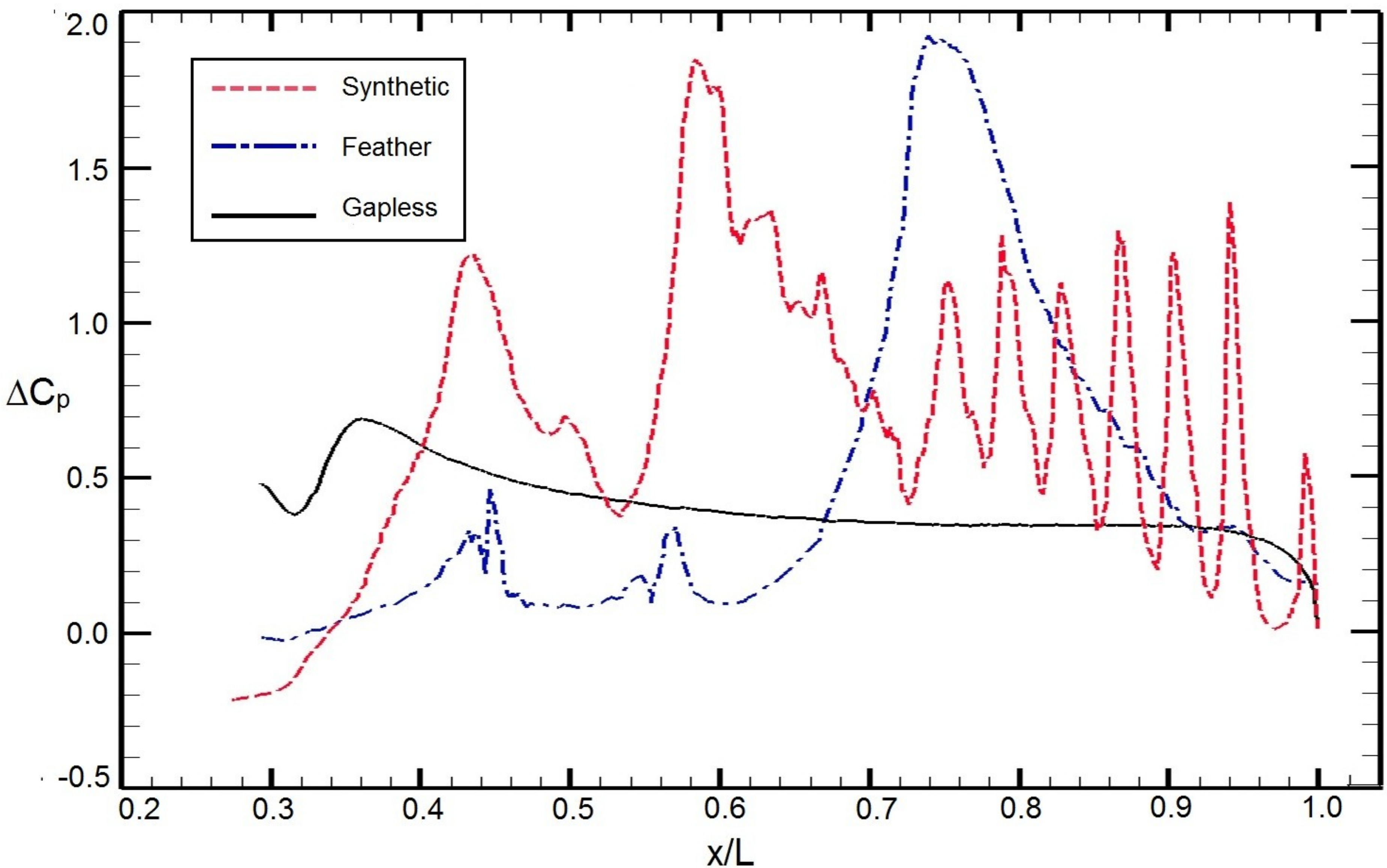
U=50 m/s, Re = 2.22 X 10^5





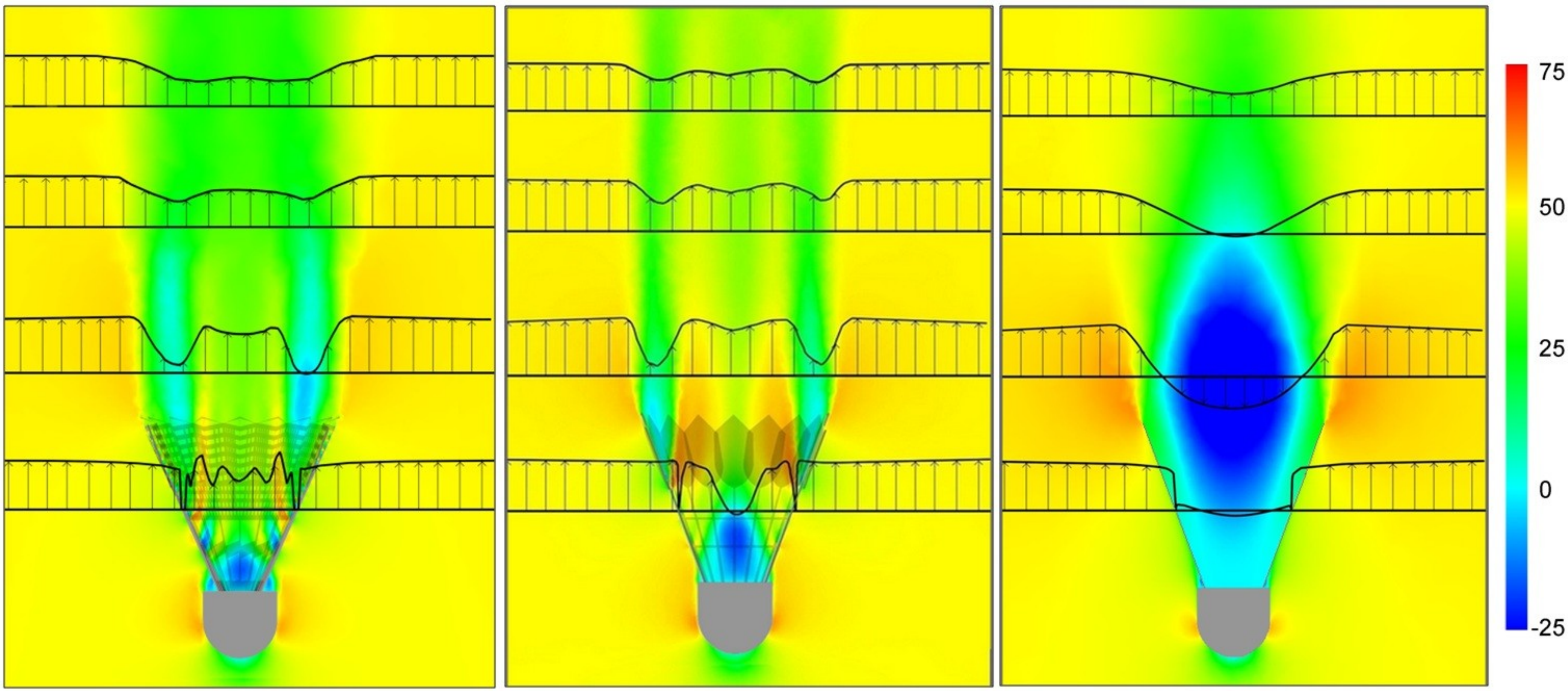
Difference In Pressure Coefficient: (Out-In)

U=50 m/s, Re = 2.22 X 10^5



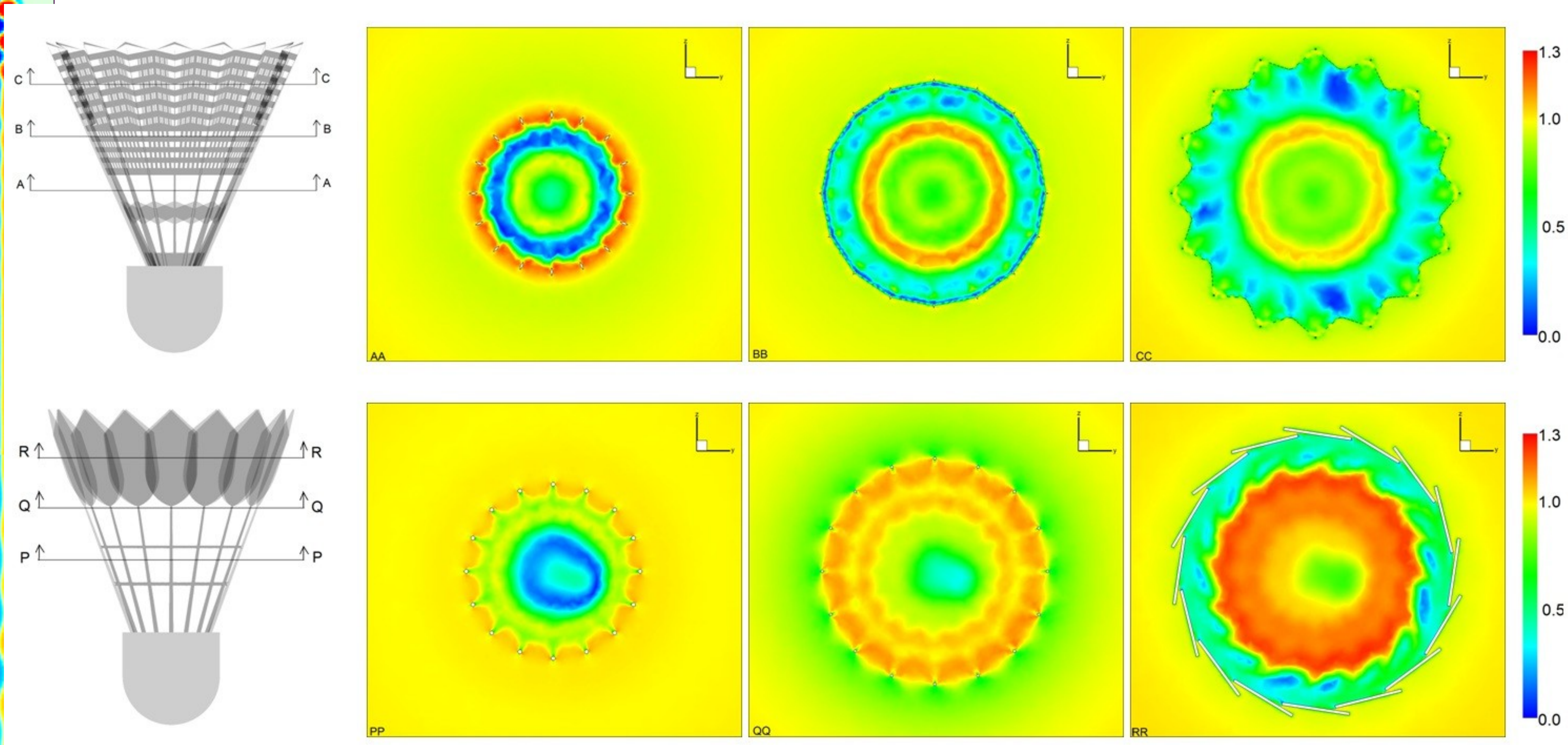
Velocity Profiles

U=50 m/s, Re = 2.22 X 10^5



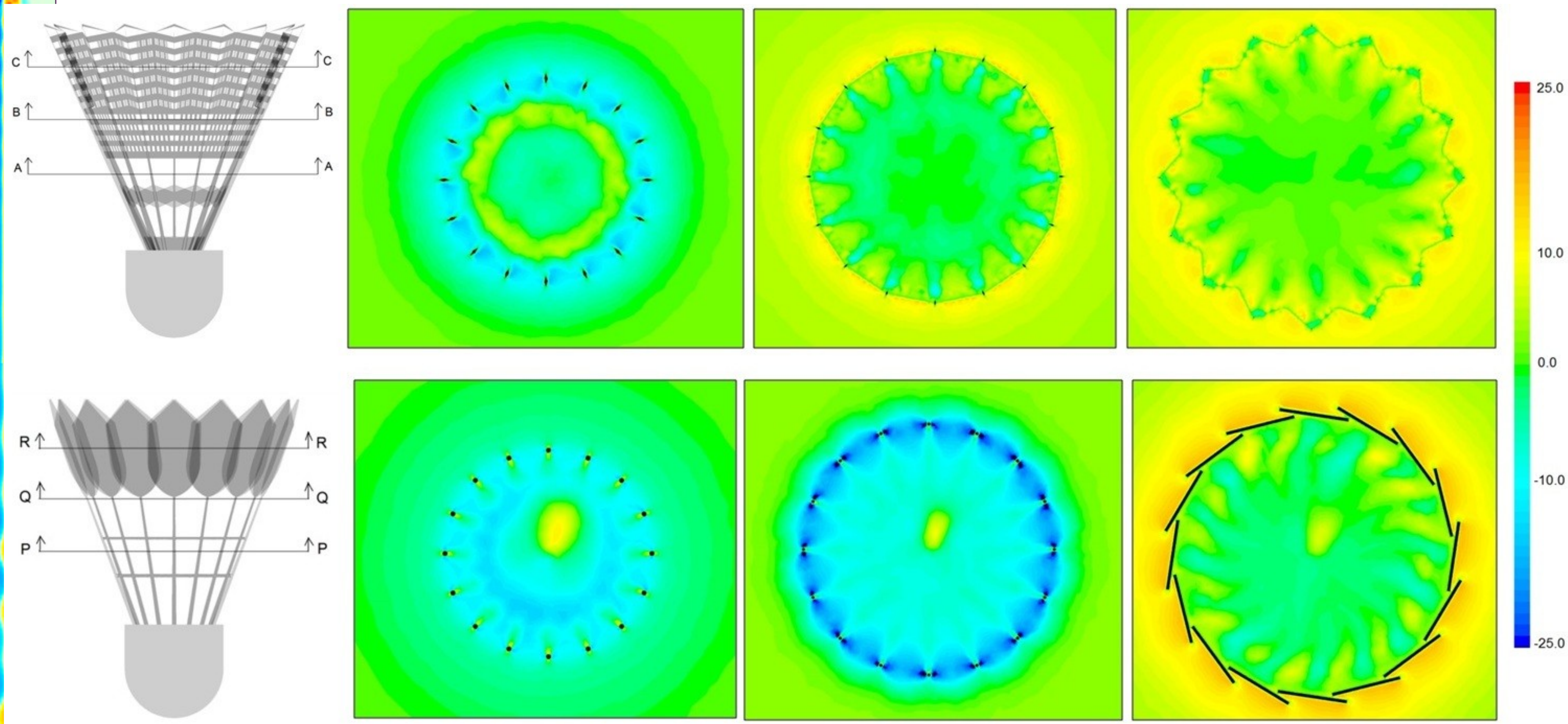
Velocity Magnitude

U=50 m/s, Re = 2.22×10^5



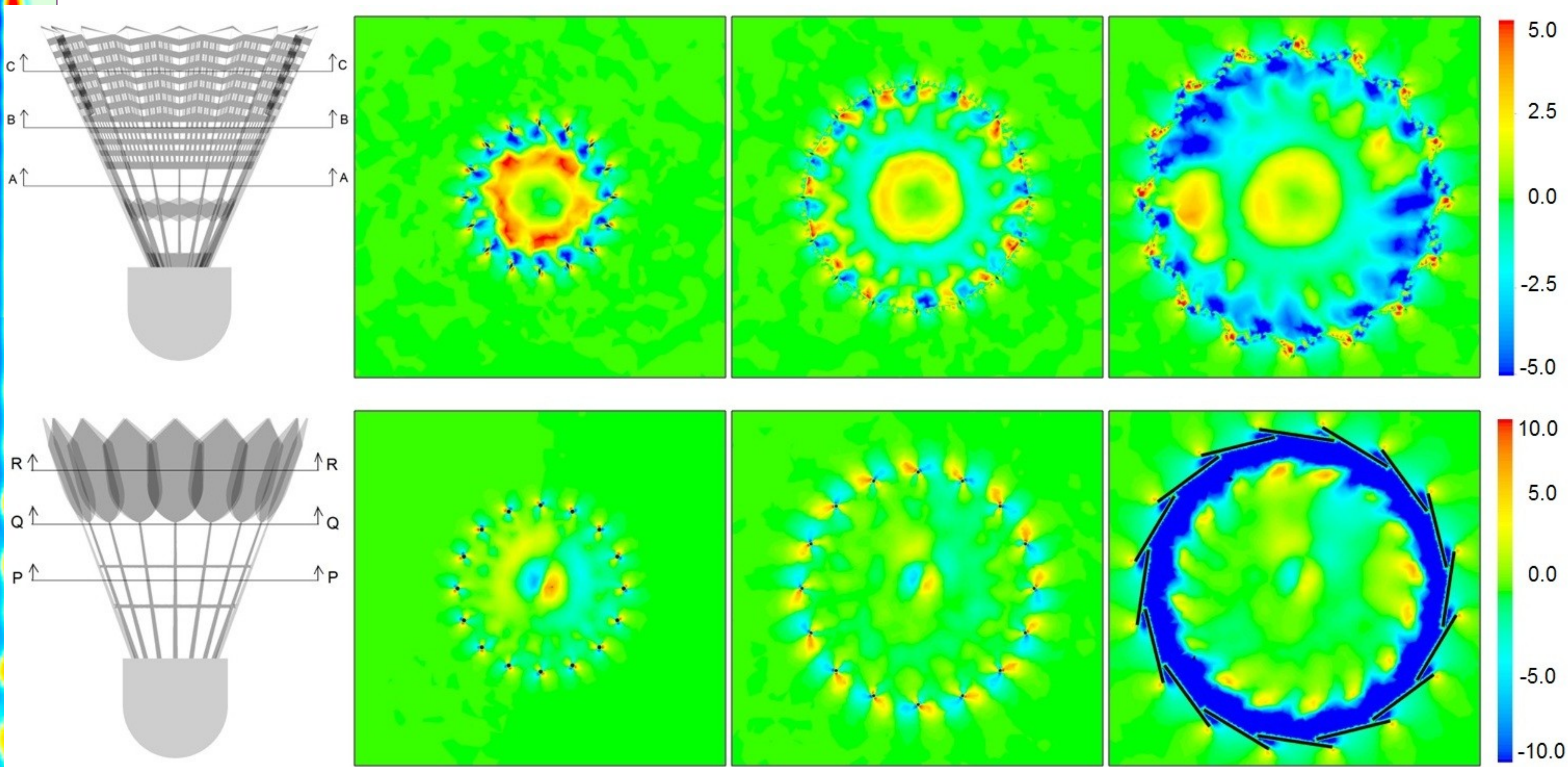
Radial Velocity

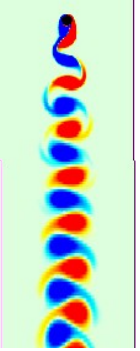
U=50 m/s, Re = 2.22 X 10^5



Tangential Velocity

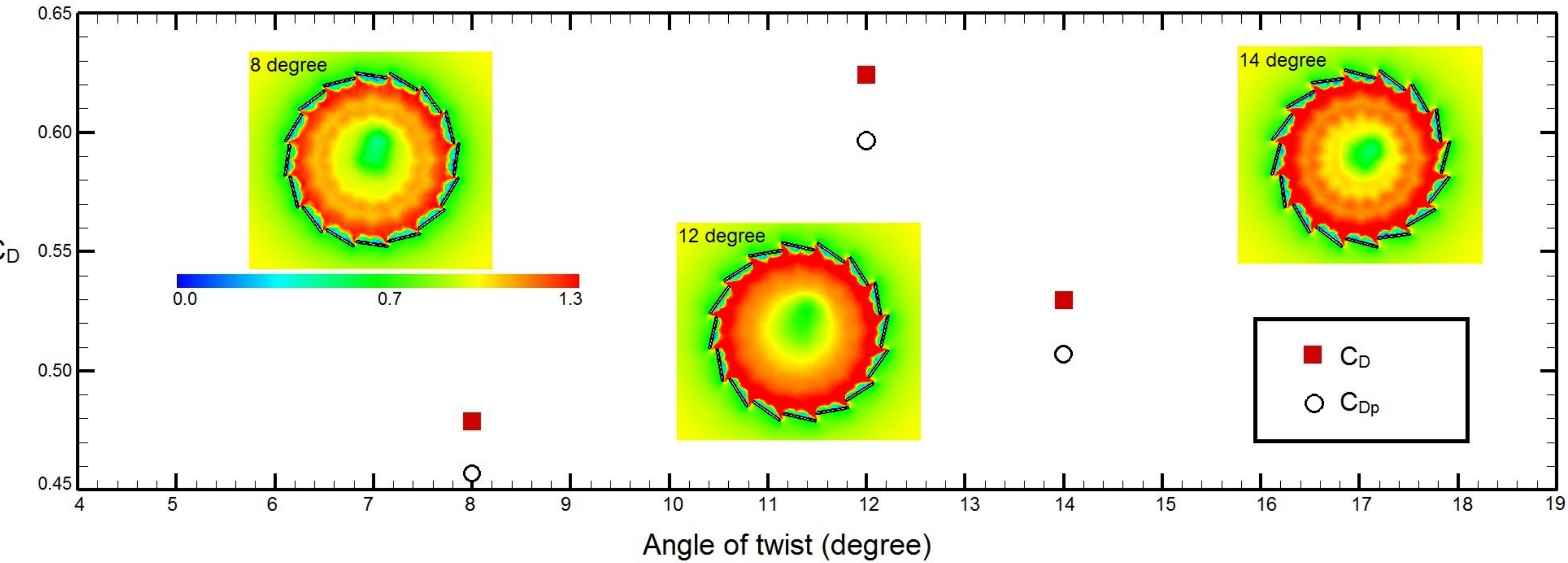
U=50 m/s, Re = 2.22 X 10^5

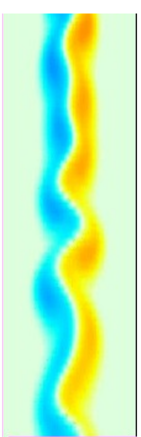




Effect of feather twist (vel magnitude)

U=50 m/s, Re = 2.22 X 10^5





Air Intake

The Concorde

Air Intake

Supplies adequate air at low speed to the engine:
flow uniformity at engine face
high efficiency (minimal losses)

Air intake of a Concorde

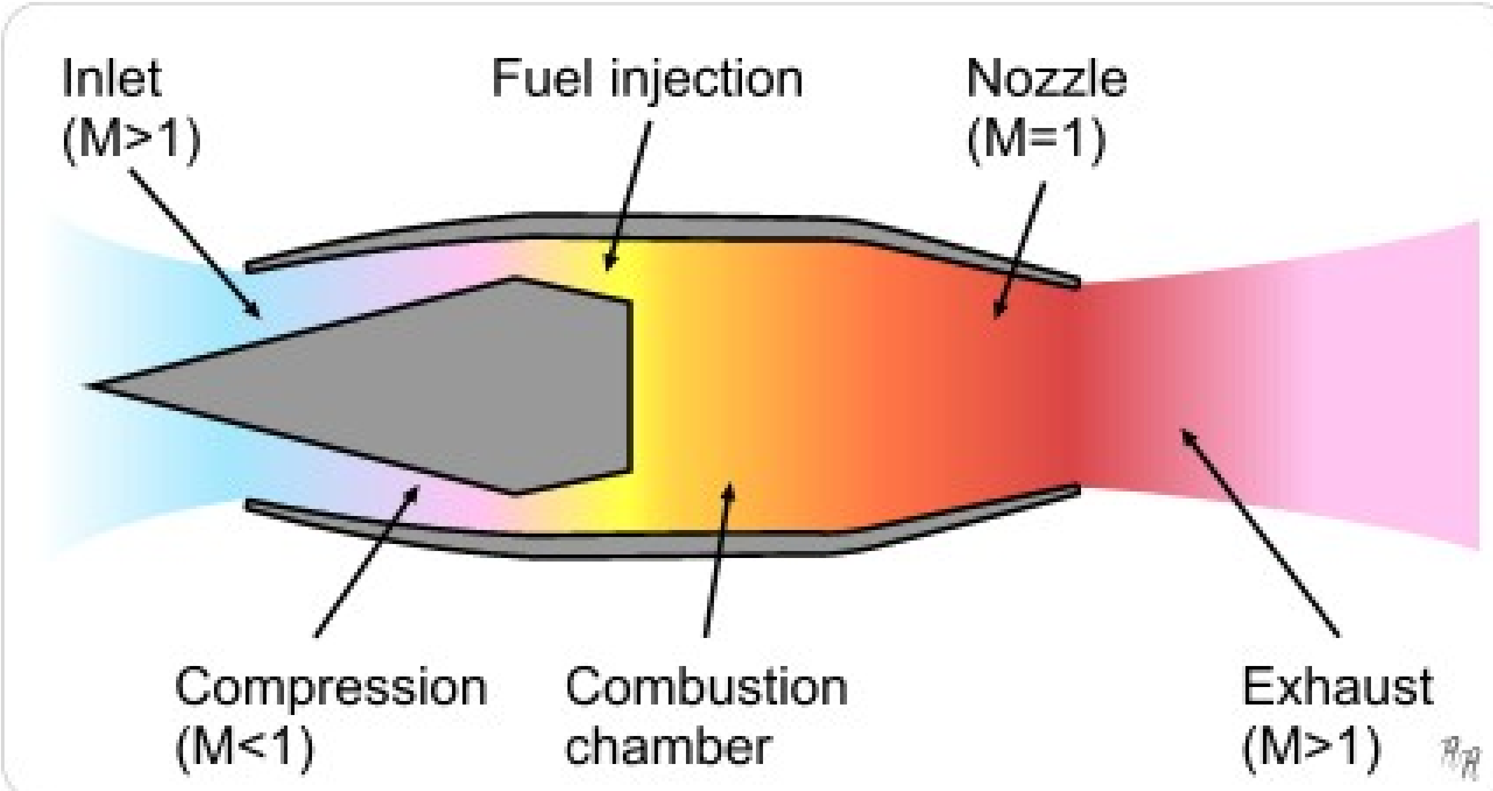
Our model

Flow in the aircraft engine is very complex

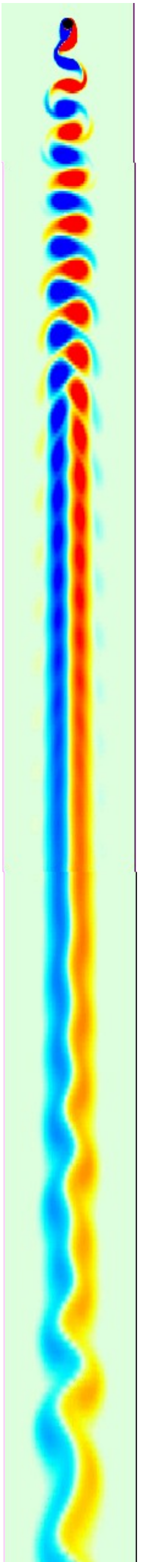
Will focus on the air intake only

Consider a Ram-jet engine; It has no turbine/compressor

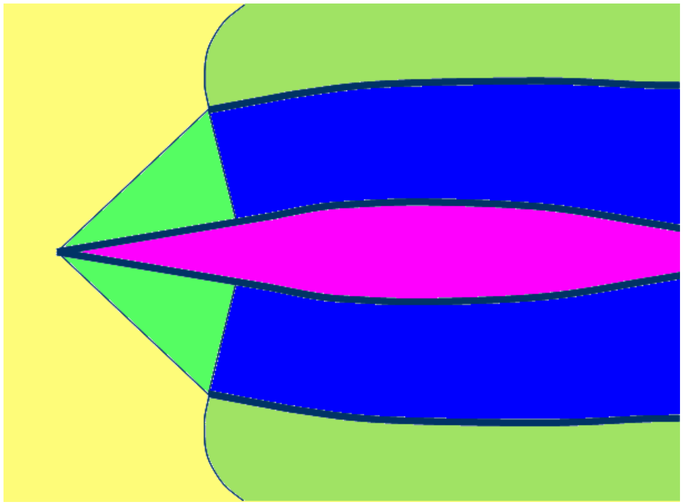
Mixed Compression Air intake



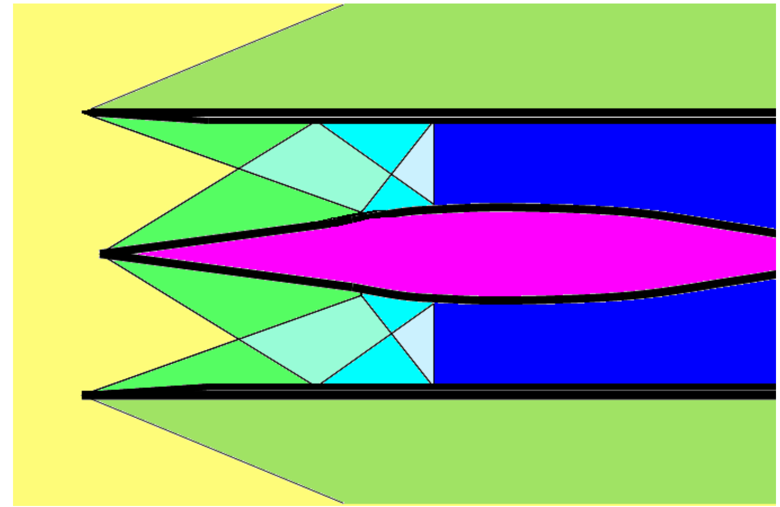
Fundamental difference in the actual working of an intake, experiments and numerics in terms of end conditions!



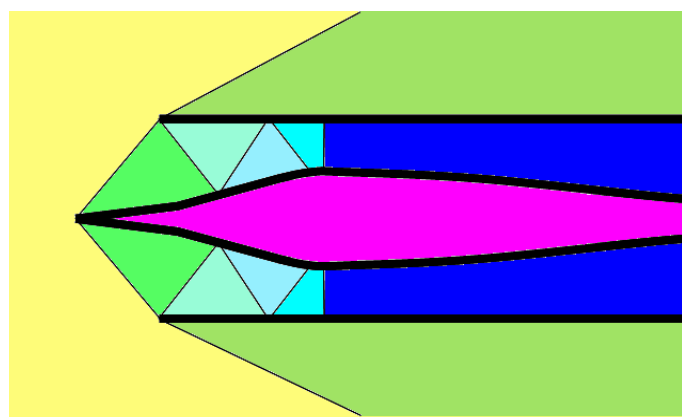
Types of Air Intakes



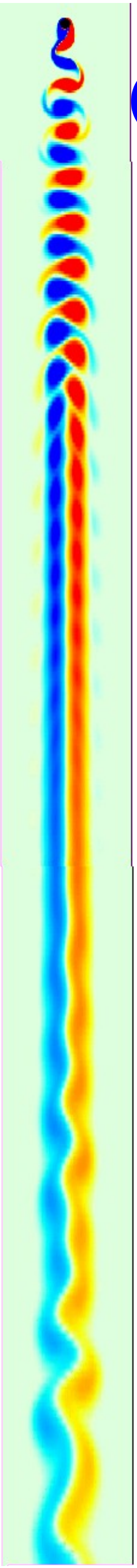
External Compression



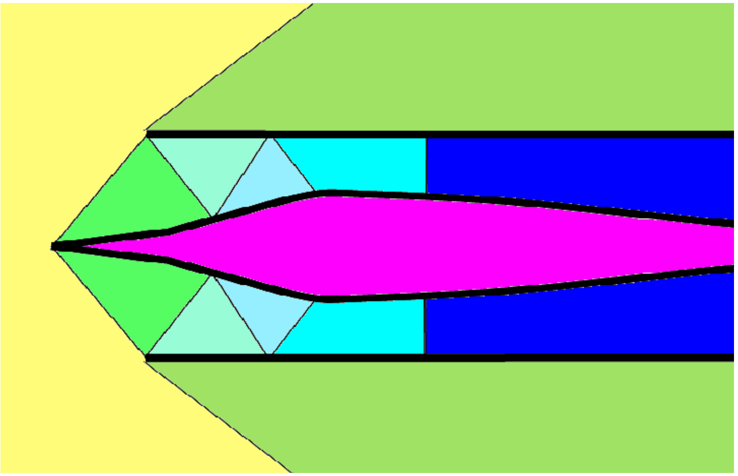
Internal Compression



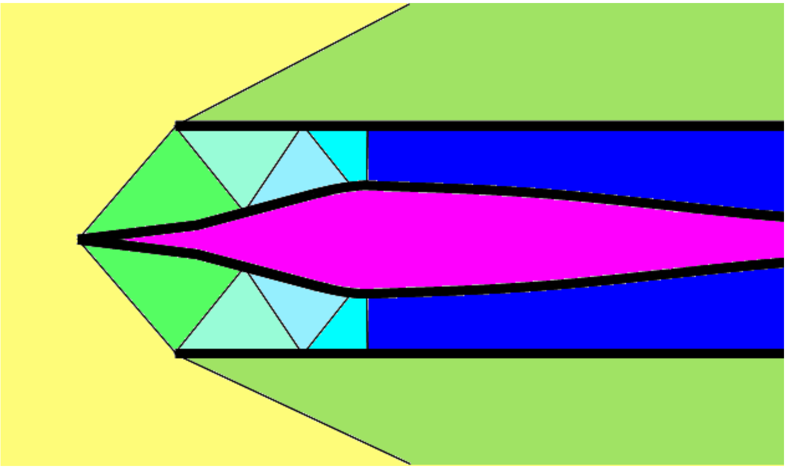
Mixed Compression



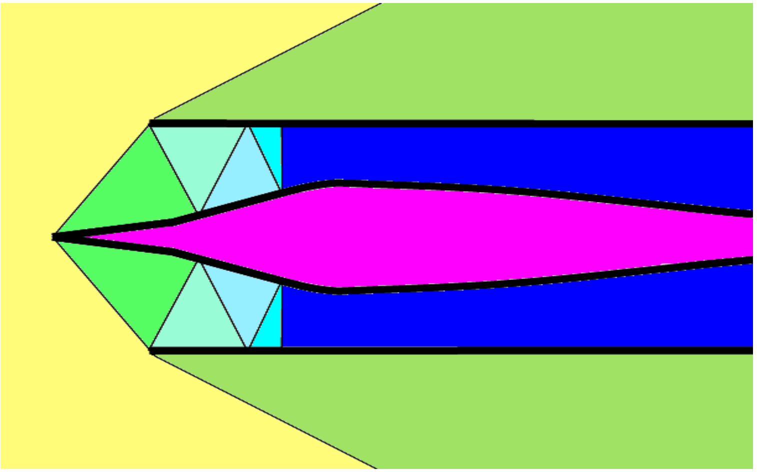
Operation of Mixed compression Air Intake



Super critical

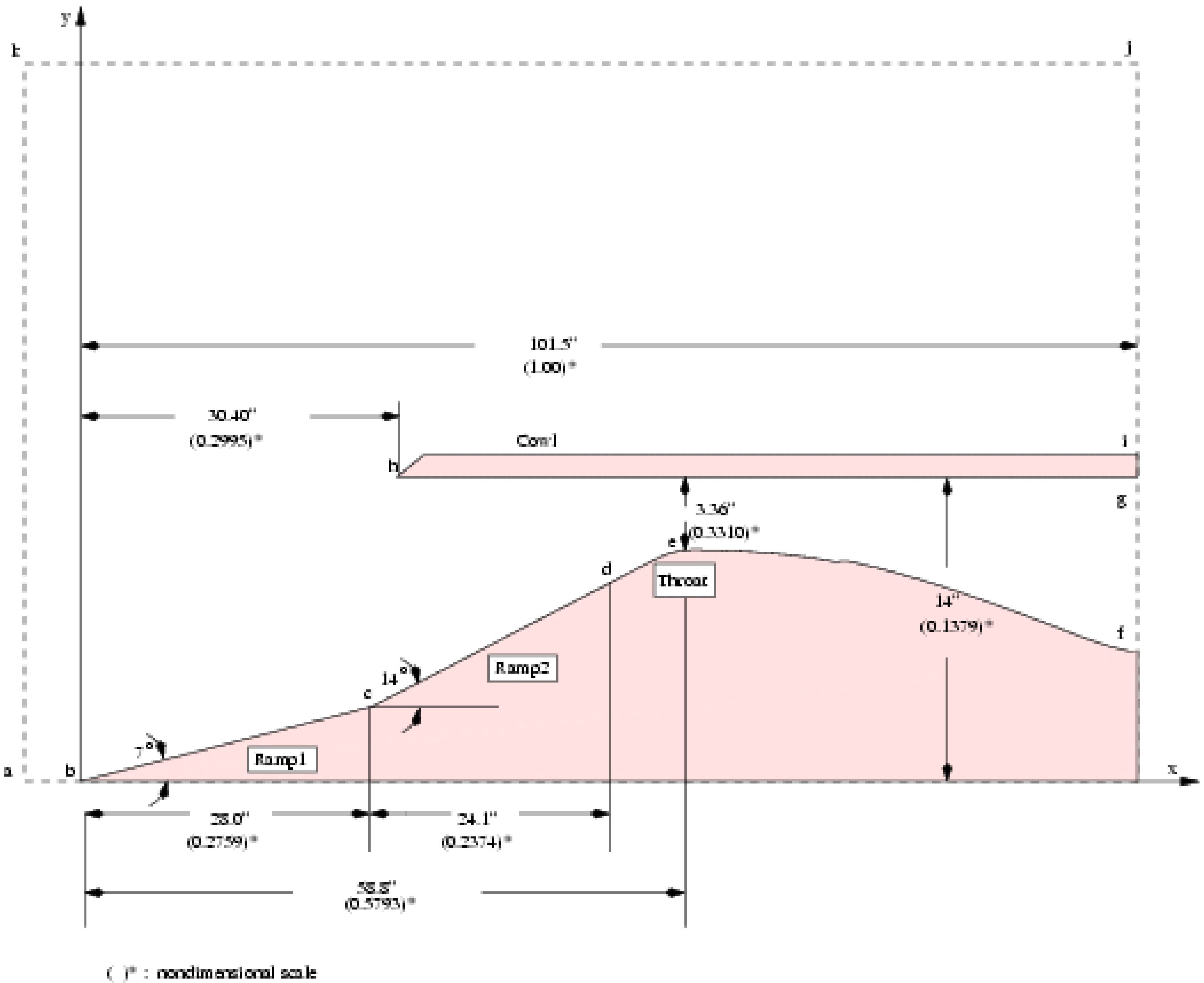


Critical

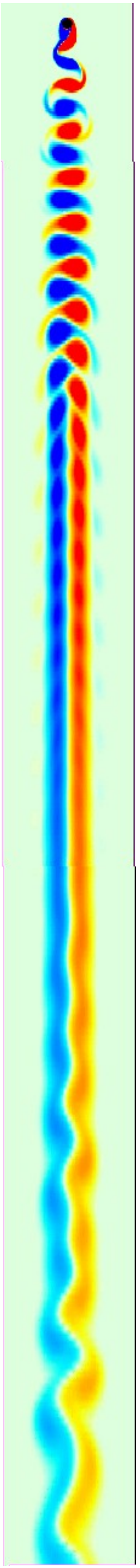


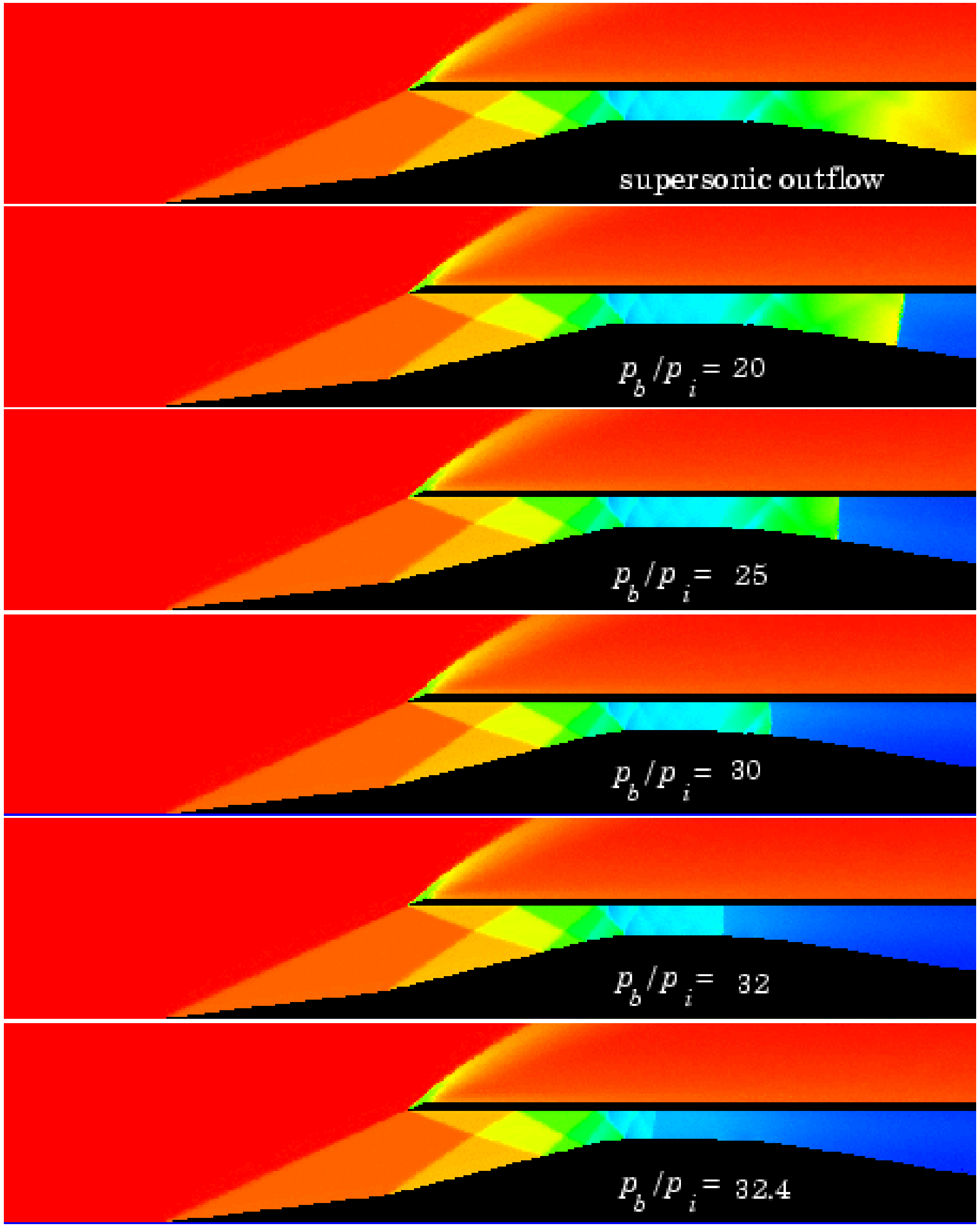
Sub critical

Mixed Compression Air intake

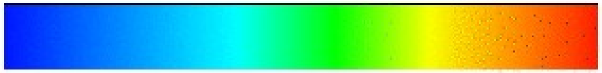


Mixed Compression Air intake: Euler



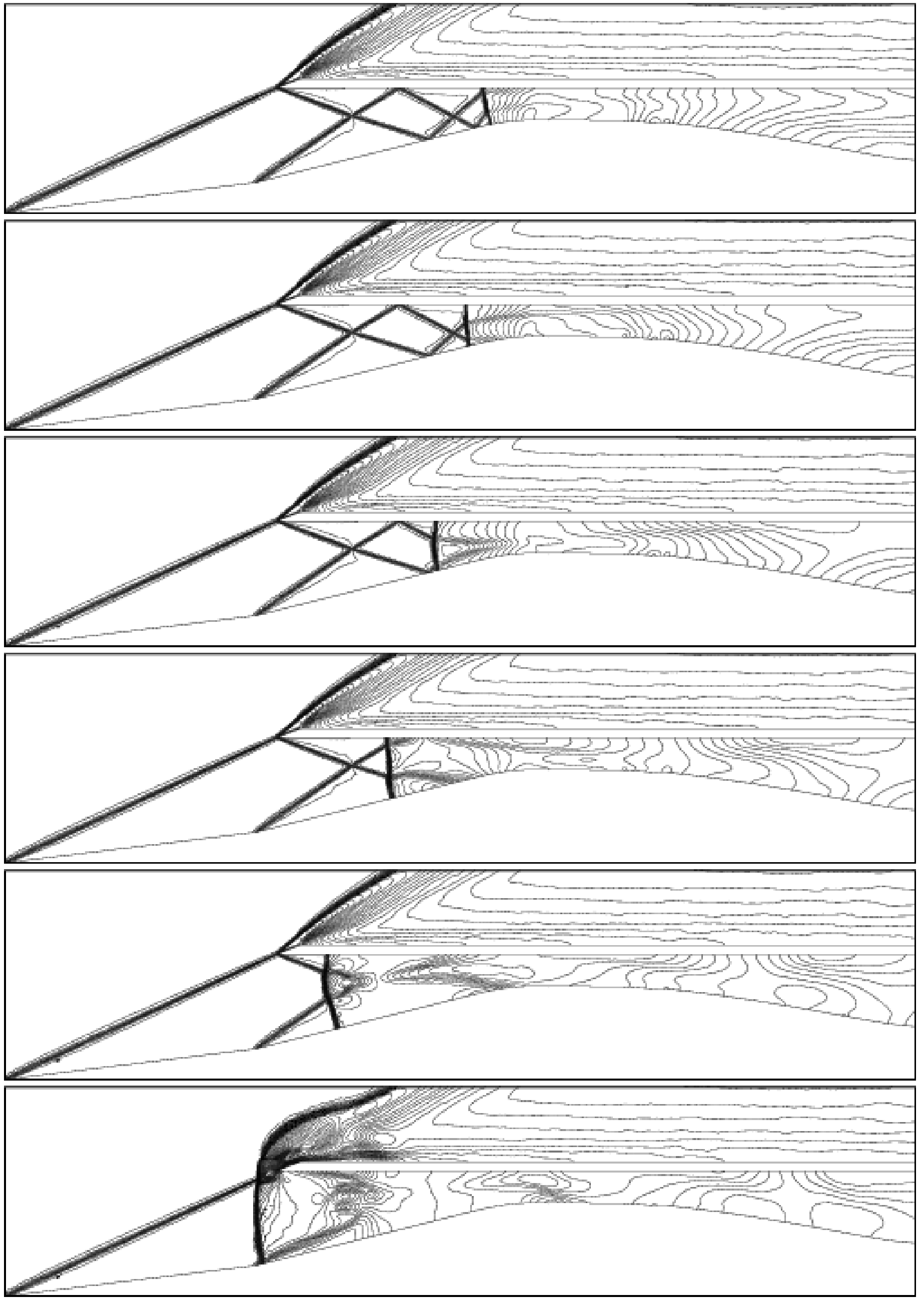


Mach number distribution for various values of back pressure.



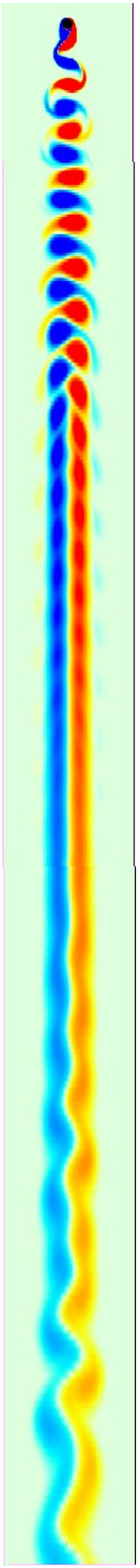
Jain & Mittal, IJNMF (2003)

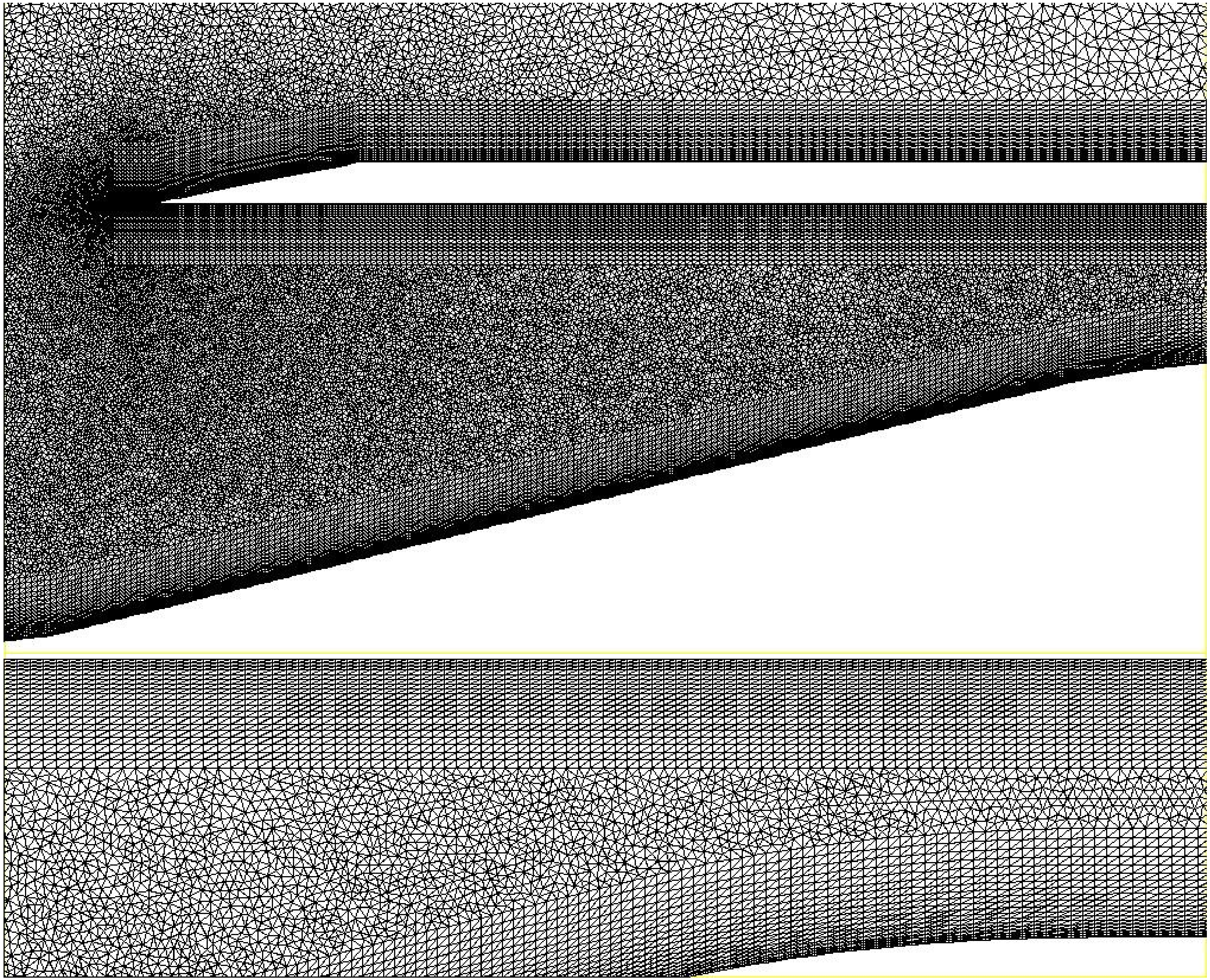
Mixed Compression Air intake: Euler



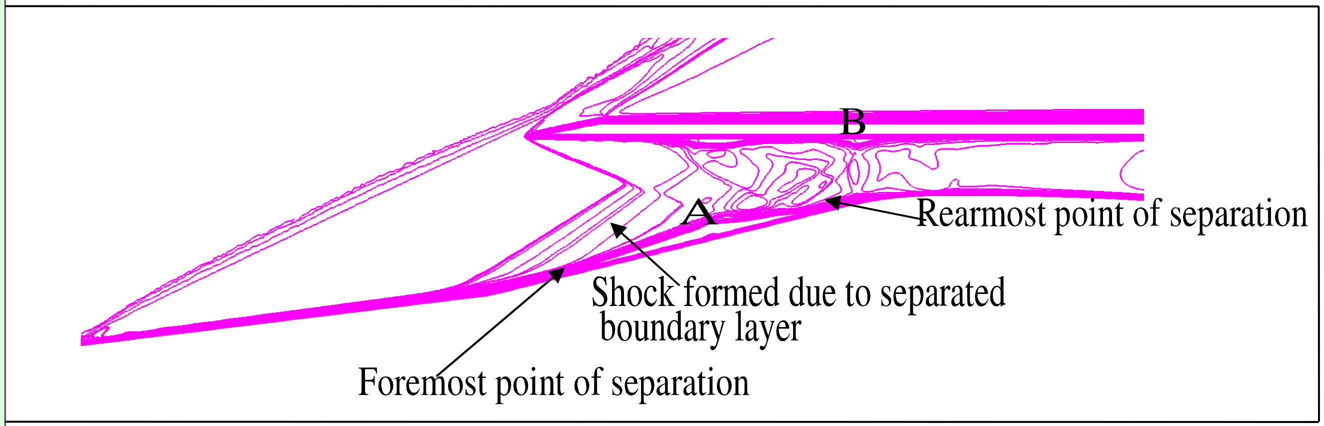
unstarting of the air intake for back pressure larger than a critical value; $p_b/p_i=32.42$

Viscous flow: the finite element mesh

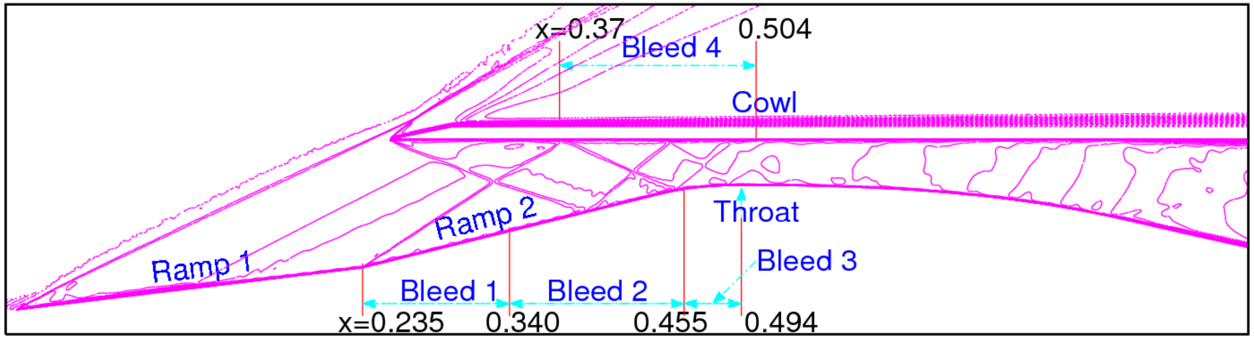




Viscous flow: bleed



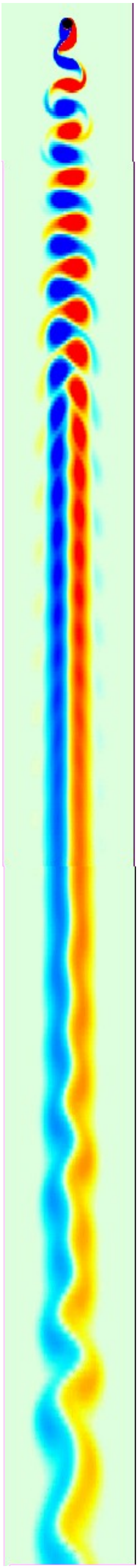
- no bleed
- unstarts

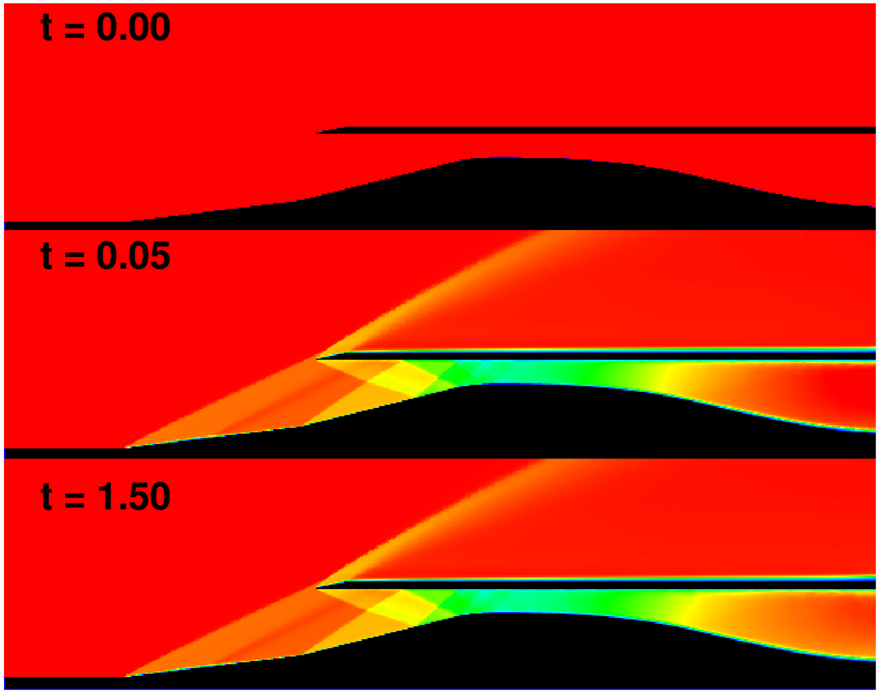


- 9 % bleed
- lower bleed
- 3.8% on cowl
- 5.2% on ramp
- starts

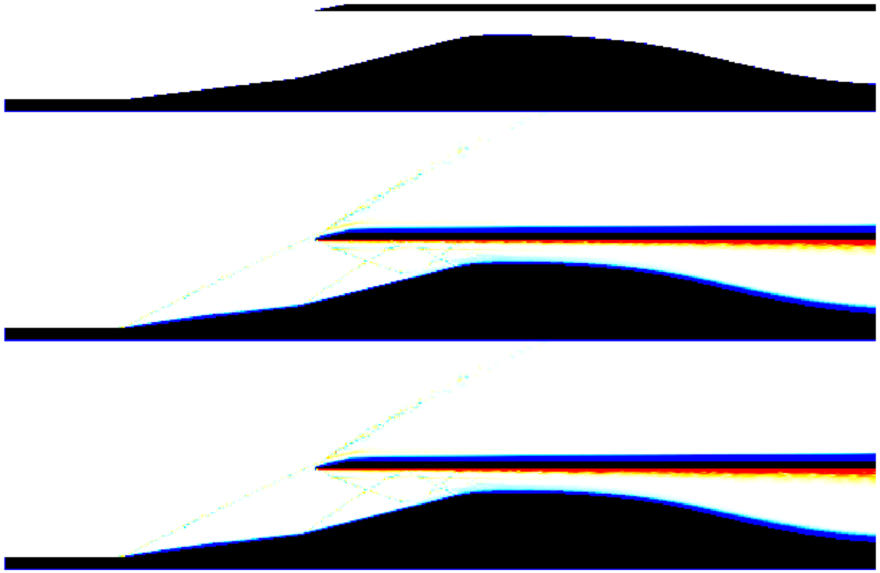
M = 3.0, Re = 10^6, 14% increase in throat area

Viscous flow: 6% bleed, M=3.0, Re=10^6
14% increase in throat area



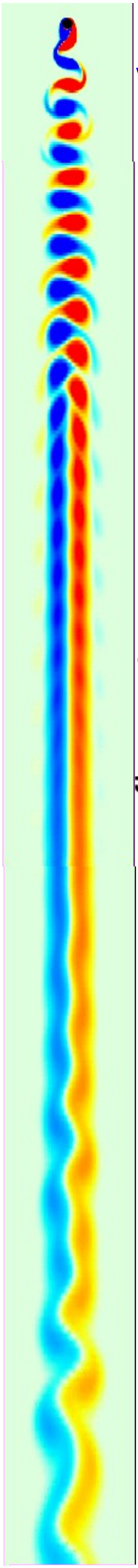


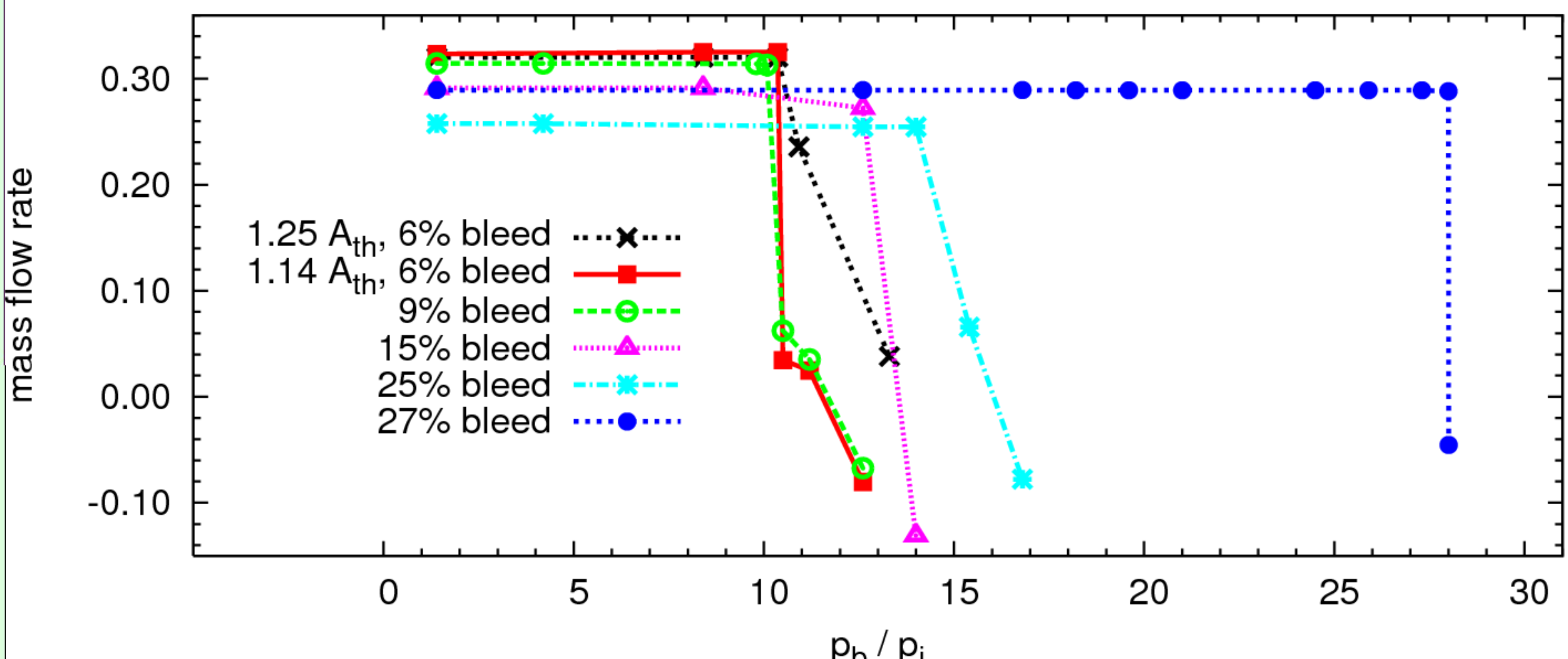
mach number



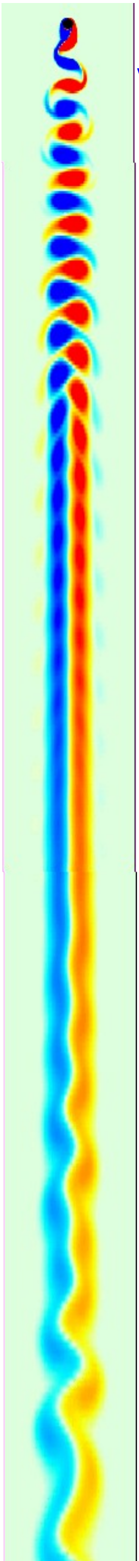
vorticity

Viscous flow: M=3.0, Re=10^6





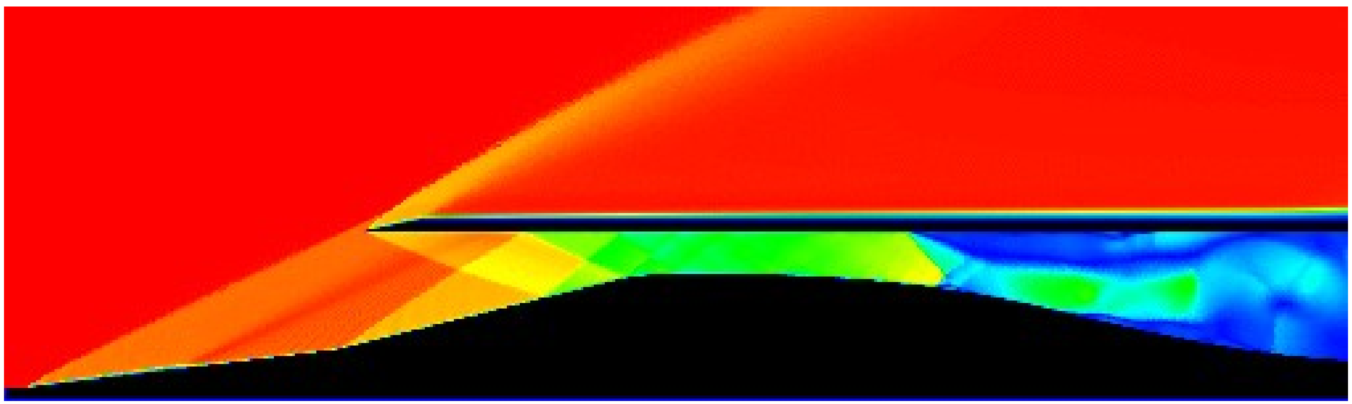
Mass flow rate at throat for various cases



Viscous flow: M=3.0, Re=10^6

14% increase in throat area, 27% bleed

$p_b / p_i = 21.0$



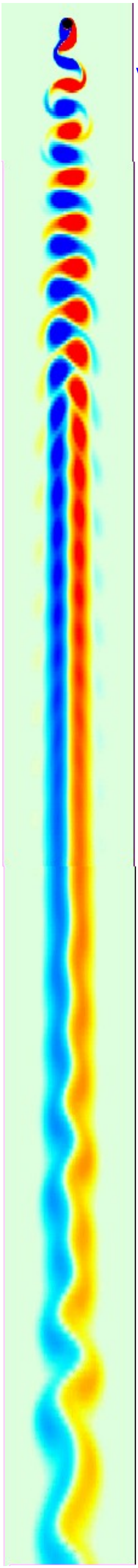
Mach number

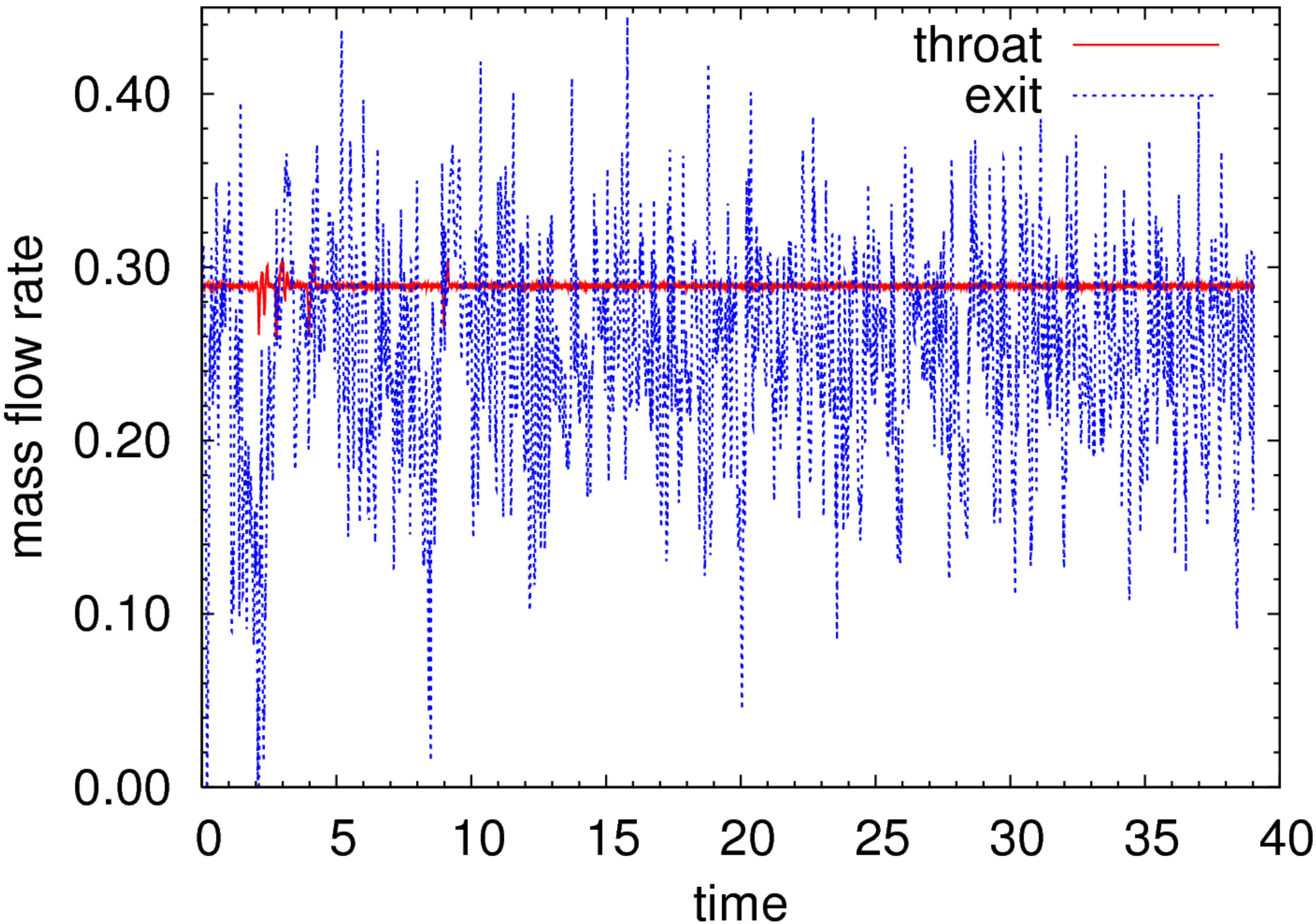
red : 3.0

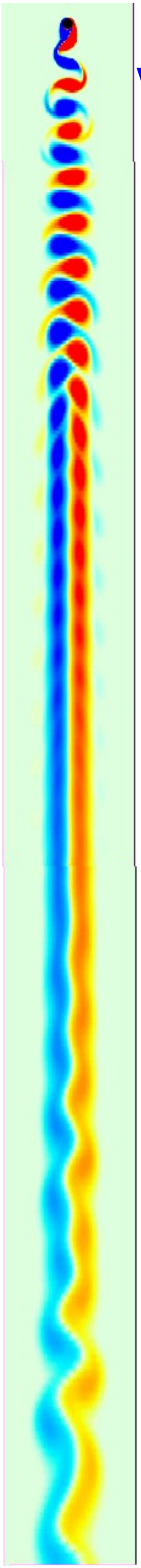
blue: 0.0

Vivek & Mittal, Jour. Of Propulsion & Power (2009)

Viscous flow: M=3.0, Re=10^6
14% increase in throat area, 27% bleed
p_b/p_i = 21.0





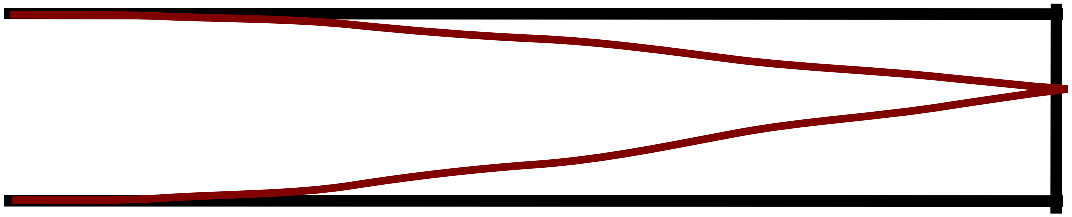


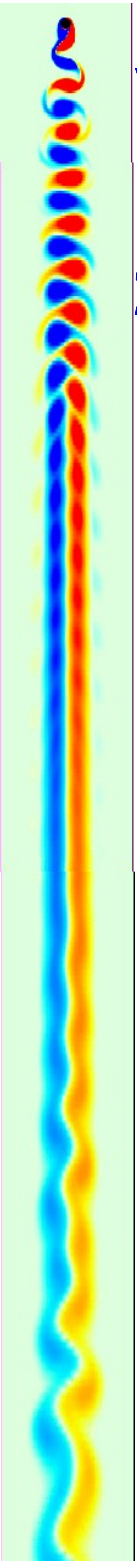
Viscous flow: $M=3.0$, $Re=10^6$

Two kinds of buzz are possible:

- **Little buzz: Ferri-Nucci type (shear layer instability)**
- **Big buzz: Dailey type (pressure/acoustic waves)**

Both are driven by superharmonics of the closed organ pipe modes of the intake

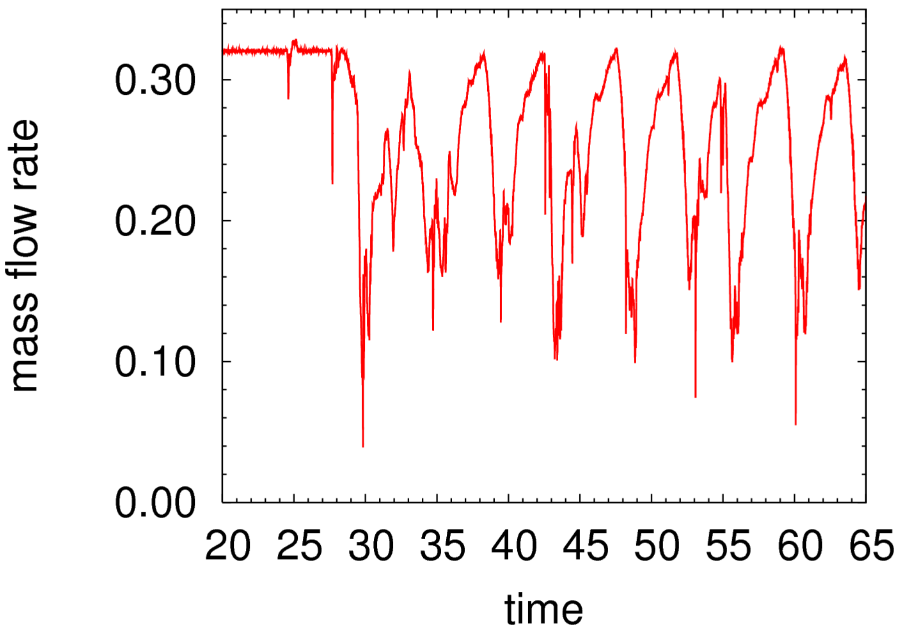


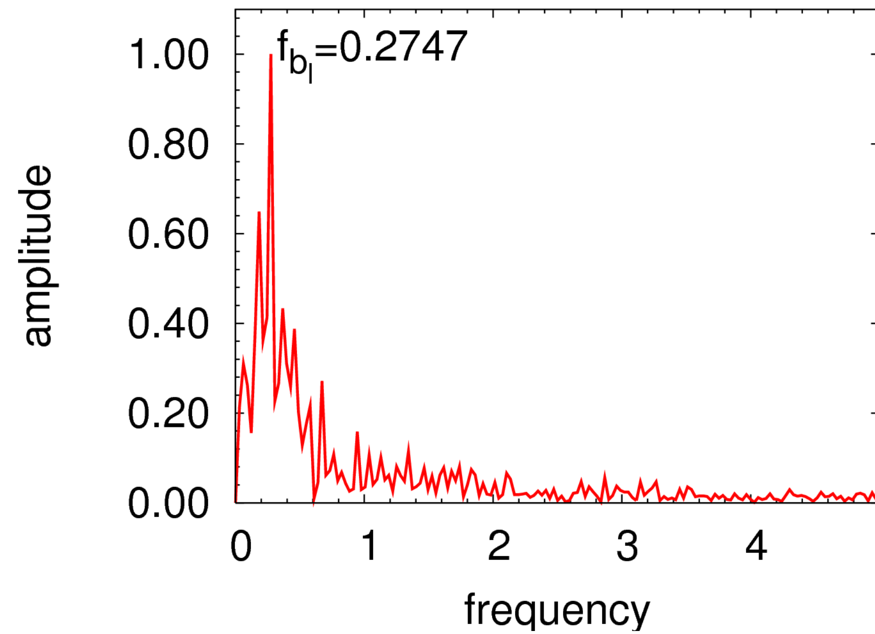


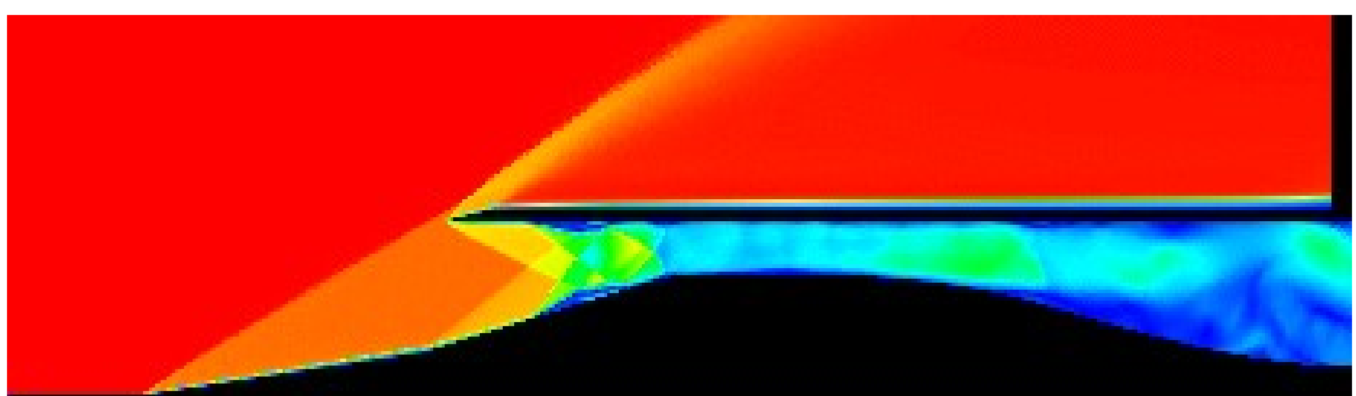
Viscous flow: $M=3.0$, $Re=10^6$

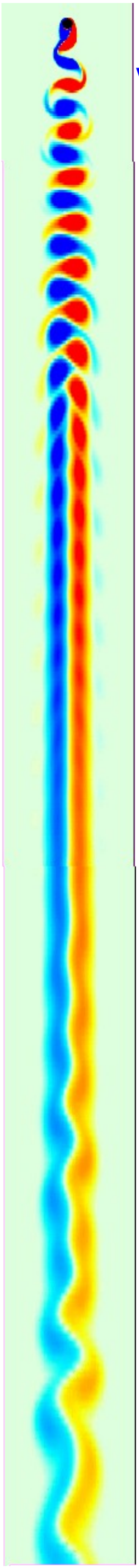
Little buzz:

25% increase in throat area, 6% bleed, p_b / p_i = 10.92





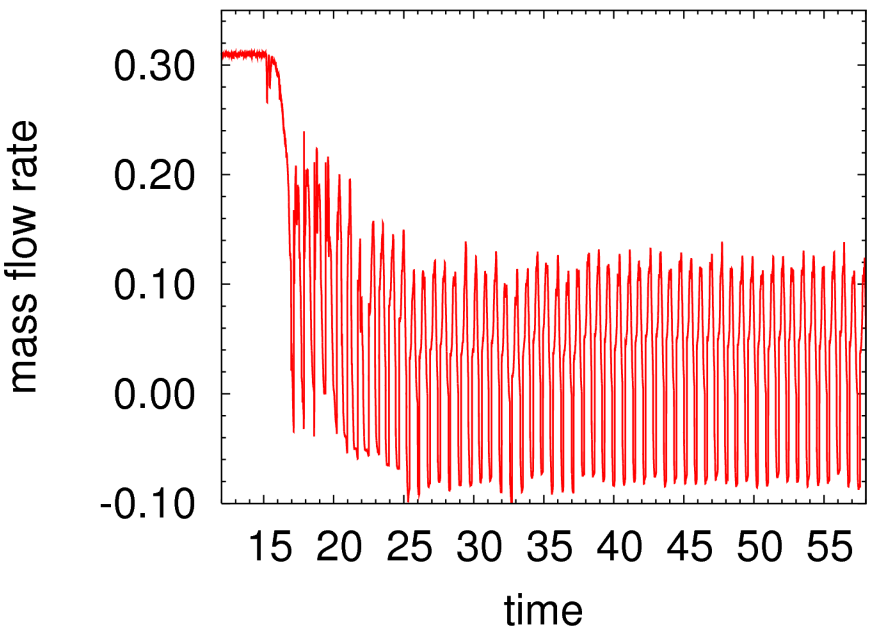


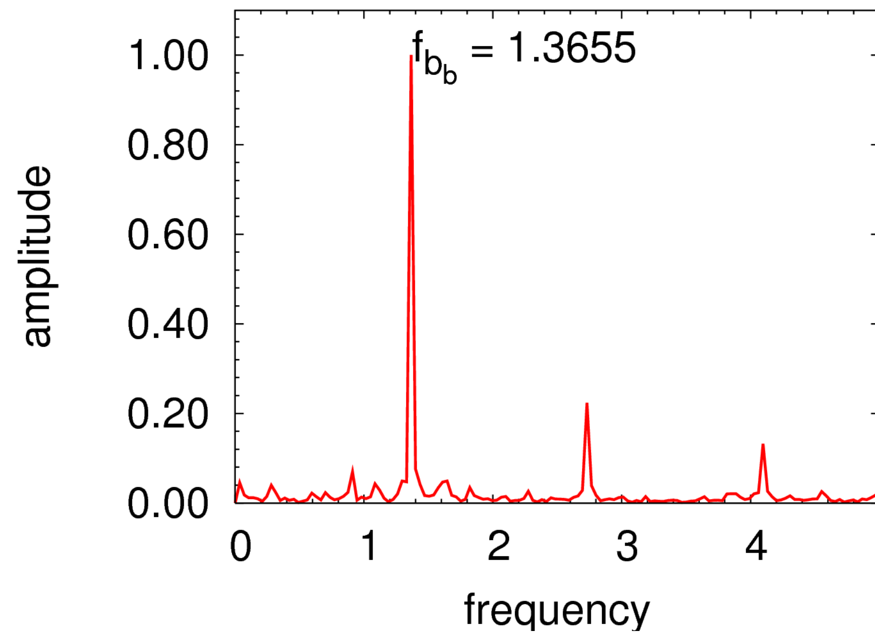


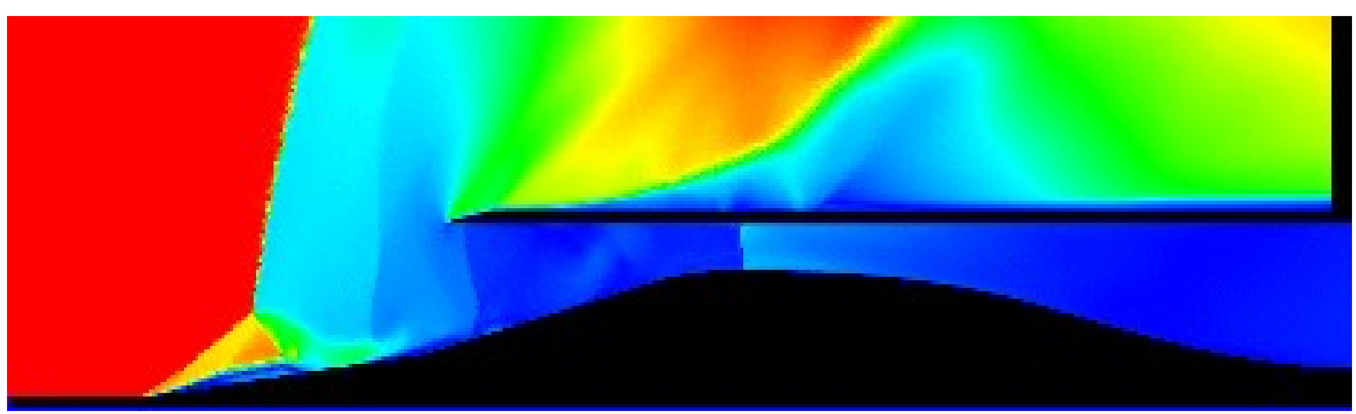
Viscous flow: M=3.0, $Re=10^6$

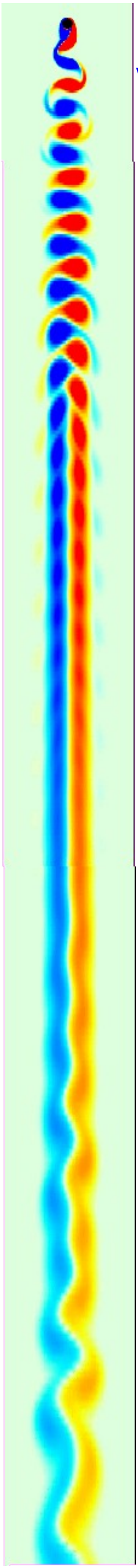
Big buzz:

14% increase in throat area, 9% bleed, $p_b / p_i = 11.2$

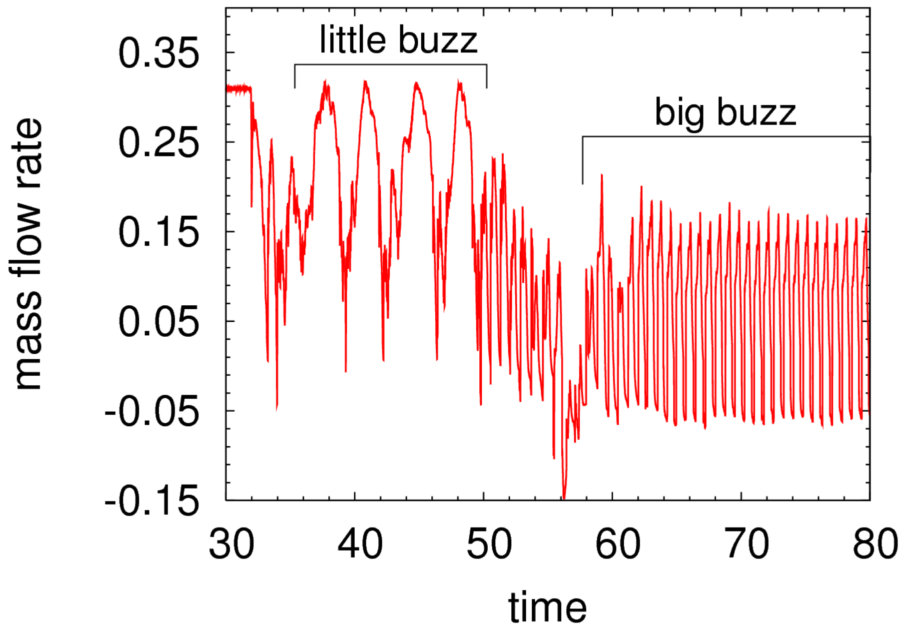


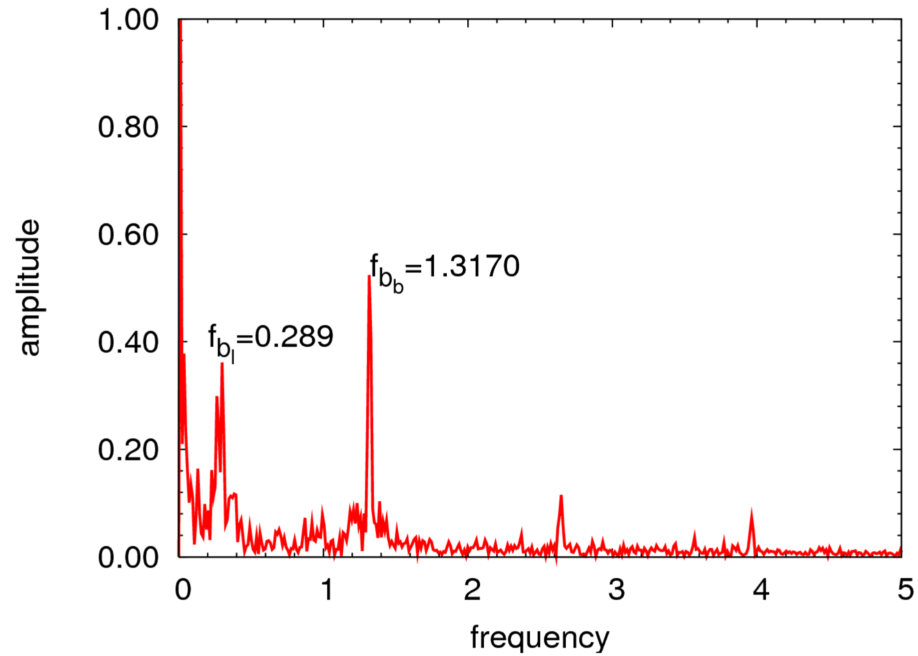


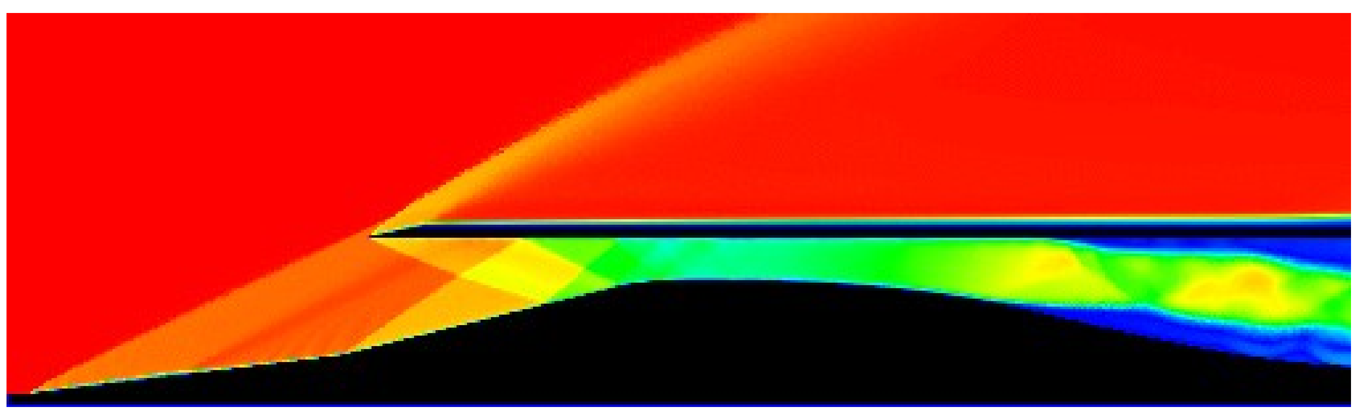




Viscous flow:M=3.0, Re=10^6
Big and Little buzz:
14% increase in throat area, 9% bleed, p_b / p_i = 10.5







Thank You

Navrose, V Murali Krushnarao Kotteda, Suresh Behara, Aekaansh Verma, Ajinkya Desai, Devashish Sharma, Sandeep Attree, Mohsin Hasan Khan, Siddharth GS, Nishith Yadav, Ankita Mittal, Amarjeet, Nischal Agrawal, Sambhav Jain, Sai Phanindra, Durgesh Vikram, Ravi Kumar